Nuclear physics input for solar models *

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Current status of the nuclear reaction cross sections of the solar p-p chain

For more details see:

http://www.sns.ias.edu/~jnb/Papers/Preprints/Solarfusion/paper.ps

$$S(E) = \sigma(E) E \exp\Big[2\pi\eta(E)\Big], \qquad \eta(E) = \frac{\mu Z_1 Z_2 e^2}{k\hbar^2}$$

1) $\underline{{}^1\mathrm{H}(p, e^+\nu_e){}^2\mathrm{H}}$

- Never been measured
- Can be calculated in "model independent way" (NN force + weak interaction)

$$S(0) = 4.00\Big(1 \pm 0.007^{+0.020}_{-0.011}\Big)\mathrm{E}{-}22 \text{ keV b}$$

2) $\underline{{}^3\mathrm{He}({}^3\mathrm{He}, 2p){}^4\mathrm{He}}$

- The only reaction that has been measured at solar energies
- The laboratory screening effect is still not fully understood
- A low-energy resonance, although unlikely, is still not completely ruled out
- Main difficulty in calculations: Coulomb three-body final state

$$S(0) = \Big(5.4 \pm 0.4\Big)\mathrm{E}{+}3 \text{ keV b}$$

3) $\underline{{}^3\mathrm{He}({}^4\mathrm{He}, \gamma){}^7\mathrm{Be}}$

- Several existing measurements

- A systematic difference between capture γ-ray and ^{7}Be activity measurements
- Precise calculations exist
- Calculations predicted the energy dependence and absolute normalization of the mirror $^3\text{H}(^4\text{He},\gamma)^7\text{Li}$ before it was measured precisely

$$S(0) = 0.53 \pm 0.5 \text{ keV b}$$

4) $\underline{^7\text{Be}(e^-,\nu_e)^7\text{Li}}$

- Never been measured (^{7}Be in a plasma)
- Believed to be precisely calculated using known weak matrix elements

$$R(^7\text{Be}+e^-) = 5.60\times10^{-9}(\rho/\mu_e)T_6^{-1/2}\Big[1+0.004(T_6-16)\Big]s^{-1}$$

5) $\underline{^7\text{Be}(p,\gamma)^8\text{B}}$

- The radiative capture measurements (many of them are old) differ by 20–30 percent in absolute normalization
- The cross section has also been measured by using the Coulomb dissociation of ^{8}B, an indirect method using radioactive beams
- Large differences among theoretical models
- The most uncertain nuclear input parameter for solar models

$$S(0) = 0.019^{+0.004}_{-0.002} \text{ keV b}$$

Previously accepted value: $S(0) = 0.0224 \pm 0.0021$ keV b

Model and computational details

Microscopic **Resonating Group Model** (RGM) with two/three-cluster dynamics for

$$
\begin{aligned}
{}^6\mathrm{He} &= \alpha + n + n \\
{}^6\mathrm{Be} &= \alpha + p + p \\
{}^7\mathrm{Li} &= \{\alpha + t, \alpha + d + n\} \\
{}^7\mathrm{Be} &= \{\alpha + h, \alpha + d + p\} \\
{}^8\mathrm{Li} &= \alpha + t + n \\
{}^8\mathrm{B} &= \alpha + h + p
\end{aligned}
$$

Here $\alpha = {}^4\mathrm{He}$, $t = {}^3\mathrm{H}$, $h = {}^3\mathrm{He}$, and $d = {}^2\mathrm{H}$.

Wave functions:

$$
\Psi_2 = \sum_{L,S} \mathcal{A}\Big\{\Big[[\Phi^A\Phi^B]_S \chi_L(\boldsymbol{\rho})\Big]_{JM}\Big\}
$$

$$
\begin{aligned}
\Psi_3 &= \sum_{l_1,l_2,L,S} \mathcal{A}\Big\{\Big[[\Phi^A\Phi^B\Phi^C]_S \chi_{[l_1,l_2]L}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\Big]_{JM}\Big\} \\
&= \sum_{I_{AB},I,l_2} \sum_{i=1}^{N_{AB}} \mathcal{A}\Big\{\Big[[\Phi^{AB,i}_{I_{AB}}\Phi^C]_I \chi_{l_2}(\boldsymbol{\rho})\Big]_{JM}\Big\}
\end{aligned}
$$

$\mathcal{A}$ is the intercluster antisymmetrizer, Φ are translationally invariant $0s$ harmonic-oscillator shell-model states, $\boldsymbol{\rho}$ are intercluster relative coordinates, l_1, l_2, L are orbital angular momenta, S is the total spin.

χ are the unknown relative motions (the most important degrees of freedom).

Kohn-Hulthén variational method for scattering states

Search for scattering solutions of

$$\widehat{H}\Psi = E\Psi$$

with the trial function

$$\Psi^t = \sum_{i=1}^{N} c_i \varphi_i + \phi^- - S\phi^+$$

From the projection equation (variational principle)

$$\langle\, \delta\Psi^t \,|\, \widehat{H} - E \,|\, \Psi^t \,\rangle = 0$$

one gets

$$\begin{aligned}
\sum_{j=1}^{N} \langle\, \varphi_i \,|\, \widehat{H} - E \,|\, \varphi_j \,\rangle\, c_j + \langle\, \varphi_i \,|\, \widehat{H} - E \,|\, \phi^+ \,\rangle\, S &= -\langle\, \varphi_i \,|\, \phi^- \,\rangle \\
\sum_{j=1}^{N} \langle\, \phi^+ \,|\, \widehat{H} - E \,|\, \varphi_j \,\rangle\, c_j + \langle\, \phi^+ \,|\, \widehat{H} - E \,|\, \phi^+ \,\rangle\, S &= -\langle\, \phi^+ \,|\, \phi^- \,\rangle
\end{aligned}$$

φ — square integrable functions, $\phi^\pm$ — incoming/outgoing regularized Coulomb/Hankel functions. (Note: no divergent integral problems, because: $(\widehat{H} - E)\,|\, \phi^\pm \,\rangle = 0$ if $r \to \infty$). The scattering matrix, S, and c_i can be determined.

Application for many-body systems (M. Kamimura, Prog. Theor. Phys. Suppl. **62**, 236 (1977)):

If all functions in the internal region have Gaussian form, then the many-body matrix elements are reduced to analytical forms plus one-dimensional integrals.

$$\Psi^t(r) = \sum_{i=1}^{N} c_i \chi_i(r)$$

where

$$\chi_i(r) = \begin{cases} p_i \varphi_i(r) & r < r_c \\ \phi^-(k,r) - s_i \phi^+(k,r) & r > r_c \end{cases}$$

The internal and external functions are smoothly matched at r_c:

$$\begin{aligned} p_i \varphi_i(r_c) &= \phi^-(k,r_c) - s_i \phi^+(k,r_c) \\ p_i \varphi_i'(r_c) &= \phi^{-\prime}(k,r_c) - s_i \phi^{+\prime}(k,r_c) \end{aligned}$$

c_i are not independent:

$$\begin{aligned} \sum_{i=1}^{N} c_i &= 1 \\ \sum_{i=1}^{N} c_i s_i &= S \end{aligned}$$

From the projection equation one gets c_i and S.

Siegert variational method for bound states

Search for bound (resonant) solutions of

$$\widehat{H}\Psi = E\Psi$$

with the trial function

$$\Psi^t = \sum_{i=1}^{N} c_i\varphi_i + c_{N+1}\phi^+$$

From the projection equation (variational principle)

$$\langle\, \delta\Psi^t \,|\, \widehat{H} - E \,|\, \Psi^t \,\rangle = 0$$

one gets

$$\begin{aligned}
\sum_{j=1}^{N} \langle\, \varphi_i \,|\, \widehat{H} - E \,|\, \varphi_j \,\rangle\, c_j + \langle\, \varphi_i \,|\, \widehat{H} - E \,|\, \phi^+ \,\rangle\, c_{N+1} &= 0 \\
\sum_{j=1}^{N} \langle\, \phi^+ \,|\, \widehat{H} - E \,|\, \varphi_j \,\rangle\, c_j + \langle\, \phi^+ \,|\, \widehat{H} - E \,|\, \phi^+ \,\rangle\, c_{N+1} &= 0
\end{aligned}$$

$N+1$ equation for $N+2$ unknown $(c_1, c_2, ..., c_N, c_{N+1}, E)$. It only has solutions at discrete negative real (complex for resonances) energies. The solution is given by

$$D(E) = \det \begin{bmatrix}
\langle\, \varphi_1 \,|\, \widehat{H} - E \,|\, \varphi_1 \,\rangle & \langle\, \varphi_1 \,|\, \widehat{H} - E \,|\, \varphi_2 \,\rangle & \ldots & \langle\, \varphi_1 \,|\, \widehat{H} - E \,|\, \phi^+ \,\rangle \\
\langle\, \varphi_2 \,|\, \widehat{H} - E \,|\, \varphi_1 \,\rangle & \langle\, \varphi_2 \,|\, \widehat{H} - E \,|\, \varphi_2 \,\rangle & \ldots & \langle\, \varphi_2 \,|\, \widehat{H} - E \,|\, \phi^+ \,\rangle \\
\vdots & \vdots & & \vdots \\
\langle\, \phi^+ \,|\, \widehat{H} - E \,|\, \varphi_1 \,\rangle & \langle\, \phi^+ \,|\, \widehat{H} - E \,|\, \varphi_2 \,\rangle & \ldots & \langle\, \phi^+ \,|\, \widehat{H} - E \,|\, \phi^+ \,\rangle
\end{bmatrix} = 0$$

A transcendent equation for the zero of a real function (complex zero of a complex function for resonances).

A matching technique, similar to the one for scattering states, can be used in order to get analytic forms for the many-body matrix elements.

Application to solar reactions

<u>^{7}Be$(p,\gamma)^8$B</u>: Problems and results

- The reaction is highly peripheral $\Longrightarrow$ the zero energy cross section is determined by the asymptotic normalization of the ^{8}B bound state wave function: $\psi \rightarrow \bar{c} W^+(kr)/r$

- The asymptotic normalization is determined by the effective radius of the ^{7}Be–p potential

- A=7 and 8 physical observables are used to constrain the interaction radius

  "Size" of ^{7}Be (radius, quadrupole moment)

  "Size" of ^{8}B (radius, quadrupole moment)

  "Size" of the relative motion $r(^8\text{B}) - r(^7\text{Be})$

  ^{8}Li—^{8}B Coulomb displacement energy as a measure of the average size of the proton distribution

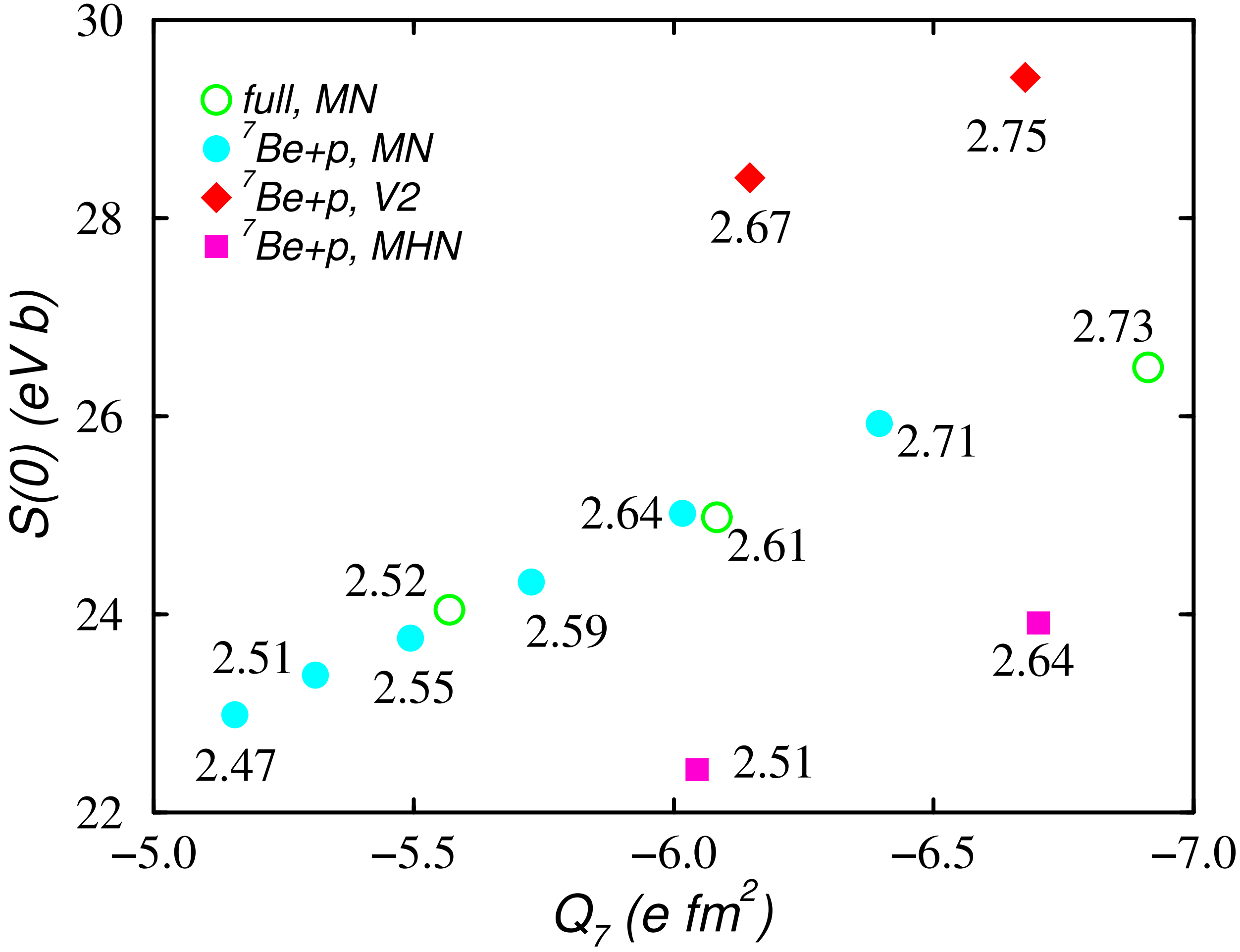
- Cross section dependence on the NN interaction

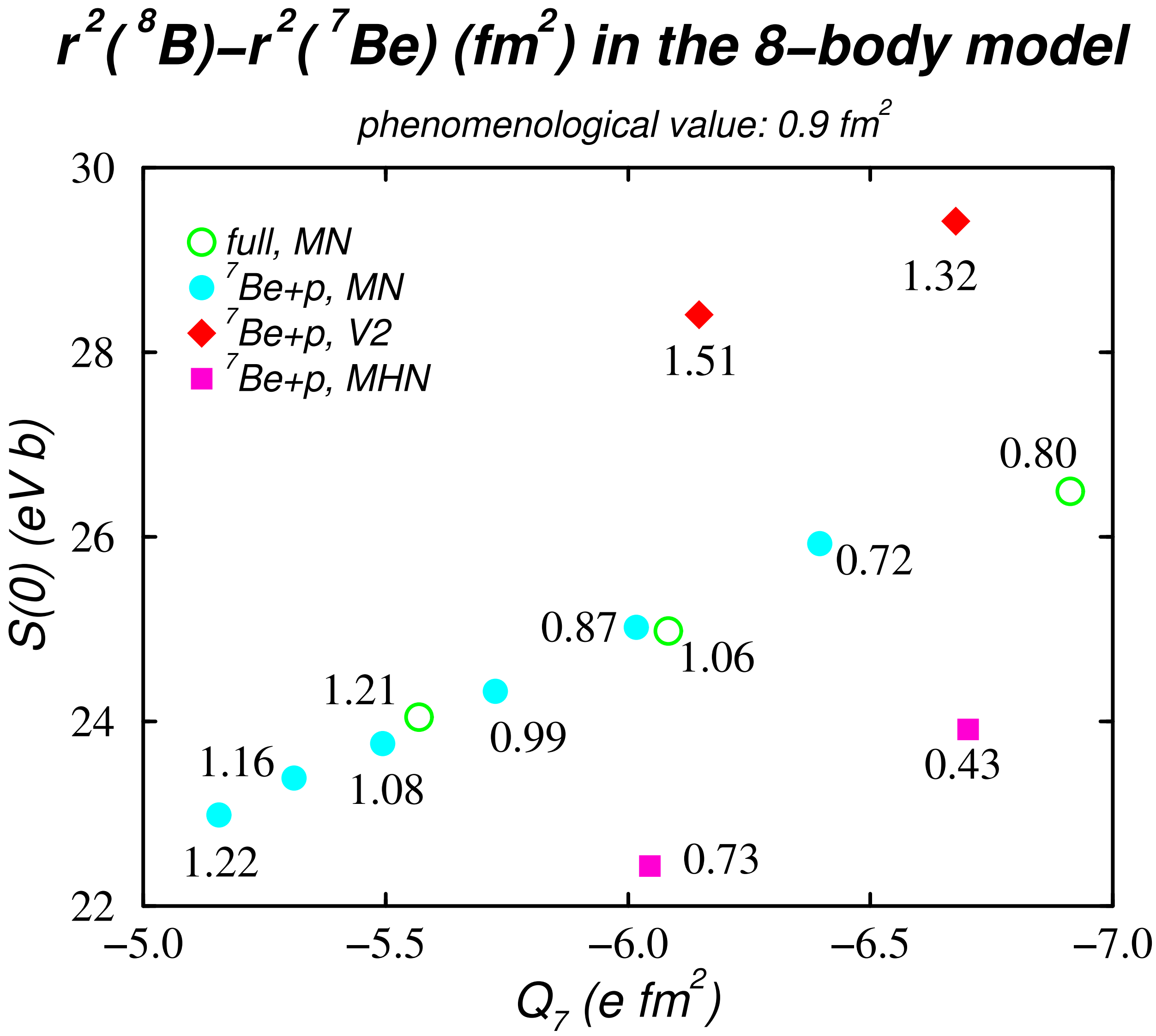
- The role of the interaction asymptotics: Yukawa vs. Gauss

- Three-body effects: where does the asymptotic region start?

Radius (fm) of ^{8}B in the 8–body model

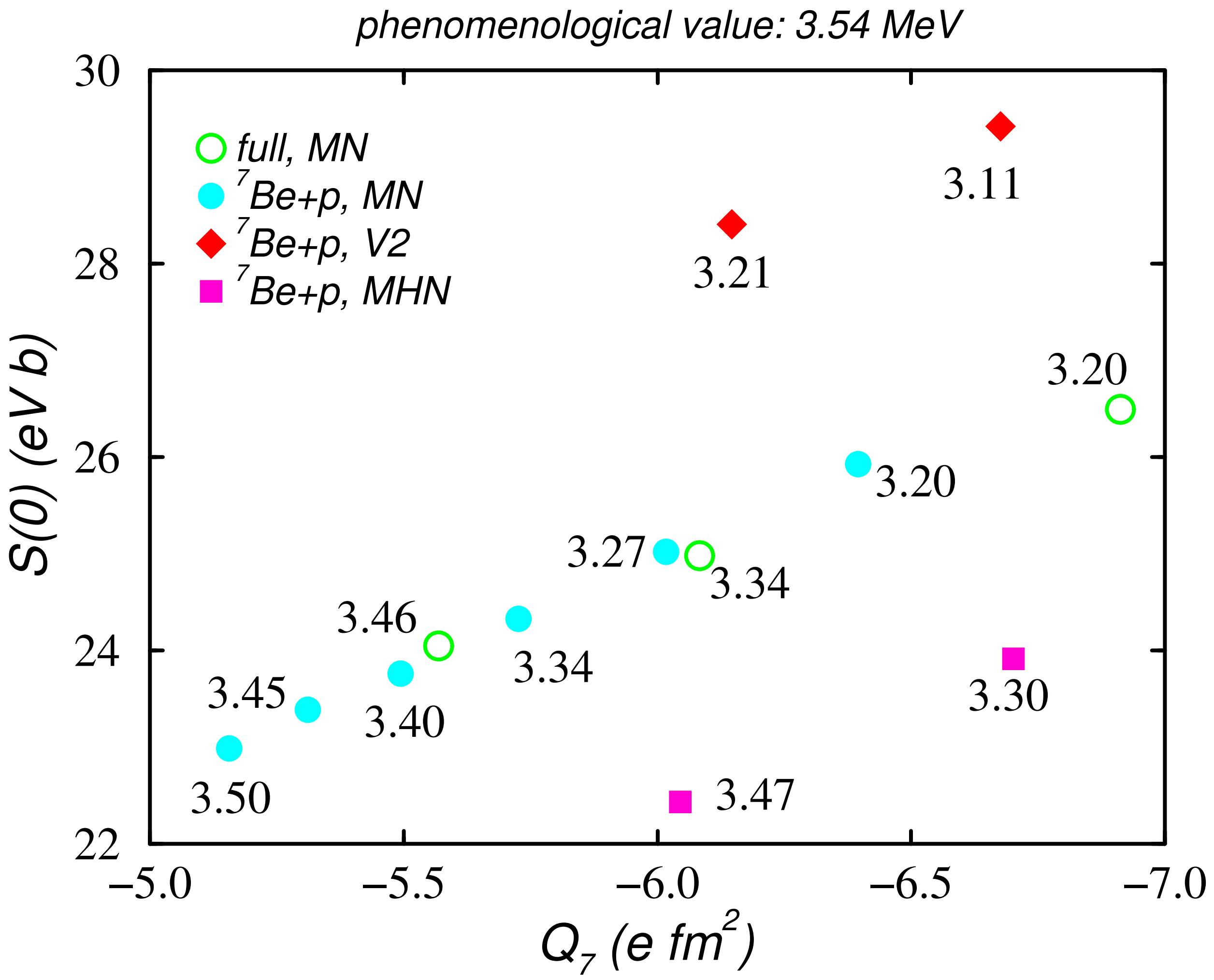
phenomenological value: 2.50±0.04 fm



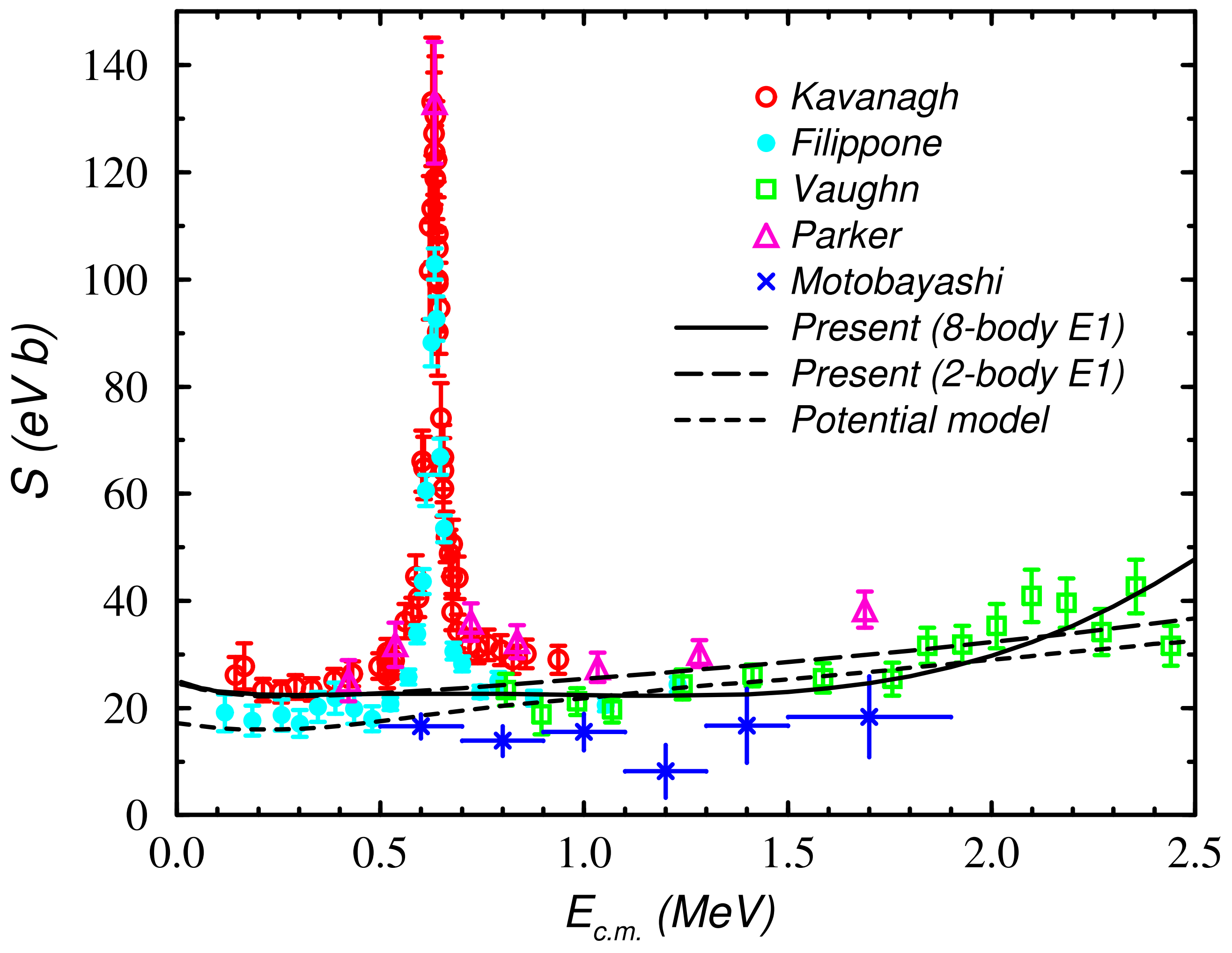


r²(⁸B)−r²(⁷Be) (fm²) in the 8−body model
phenomenological value: 0.9 fm²
full, MN
⁷Be+p, MN
⁷Be+p, V2
⁷Be+p, MHN
1.32
1.51
0.80
0.72
0.87
1.06
1.21
0.99
1.16
1.08
1.22
0.43
0.73
S(0) (eV b)
Q₇ (e fm²)
30
28
26
24
22
−5.0
−5.5
−6.0
−6.5
−7.0

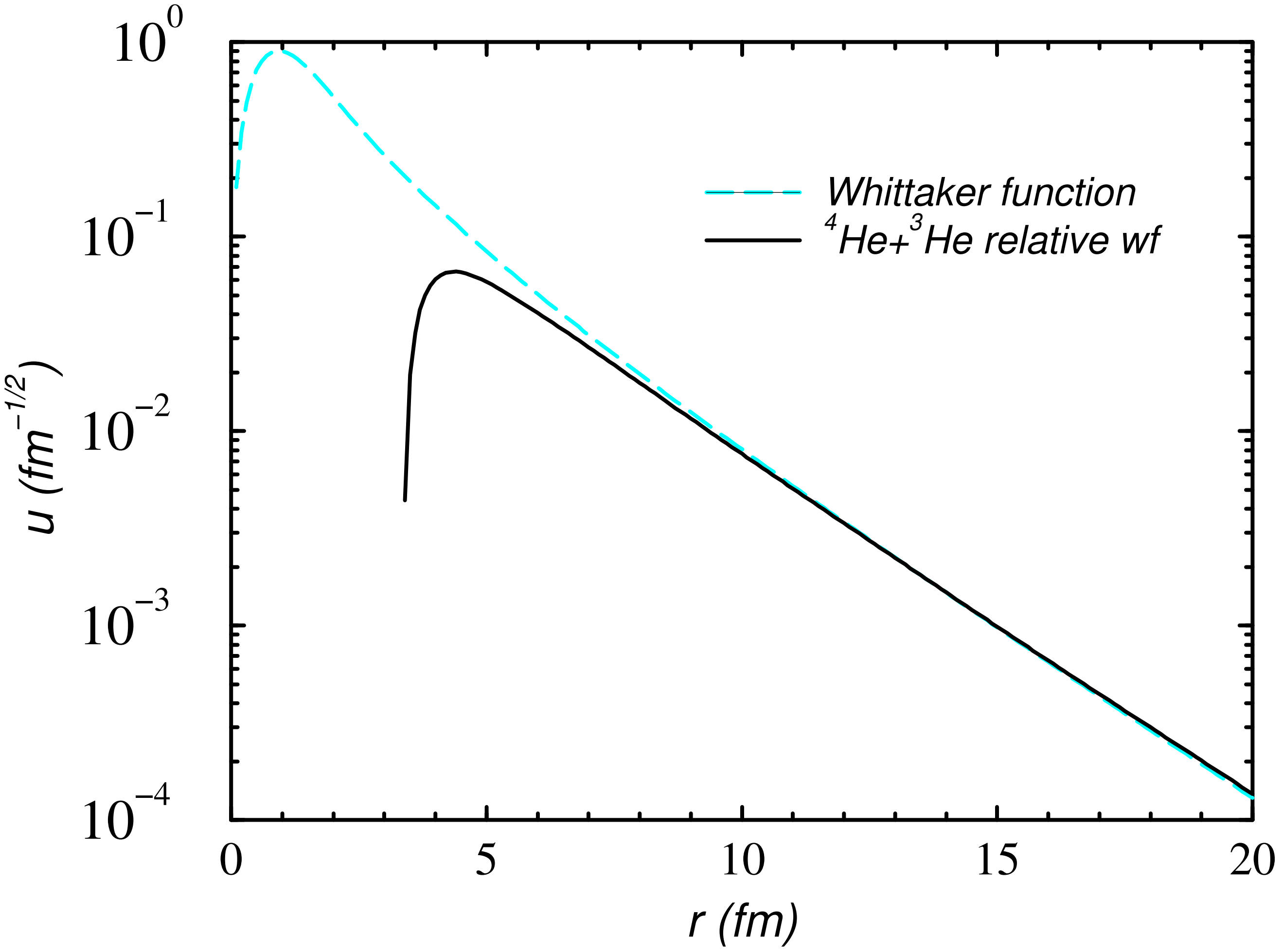
$E(^8B)-E(^8Li)$ Coulomb displacement (MeV) in the 8-body model



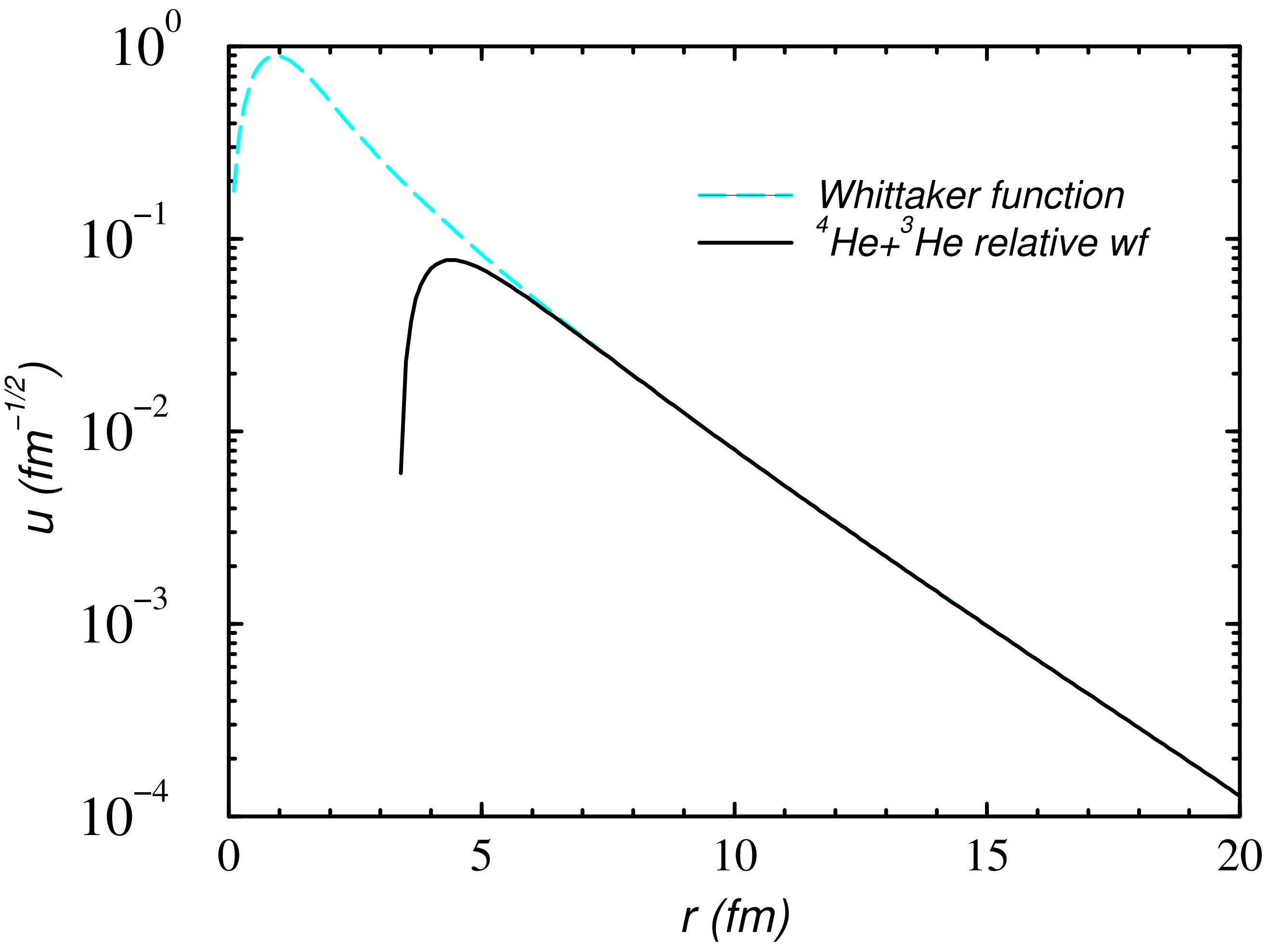
Off-shell effects in $^{7}Be(p,\gamma)^{8}B$



Asymptotic ^{7}Be wave function (with ^{6}Li+p)

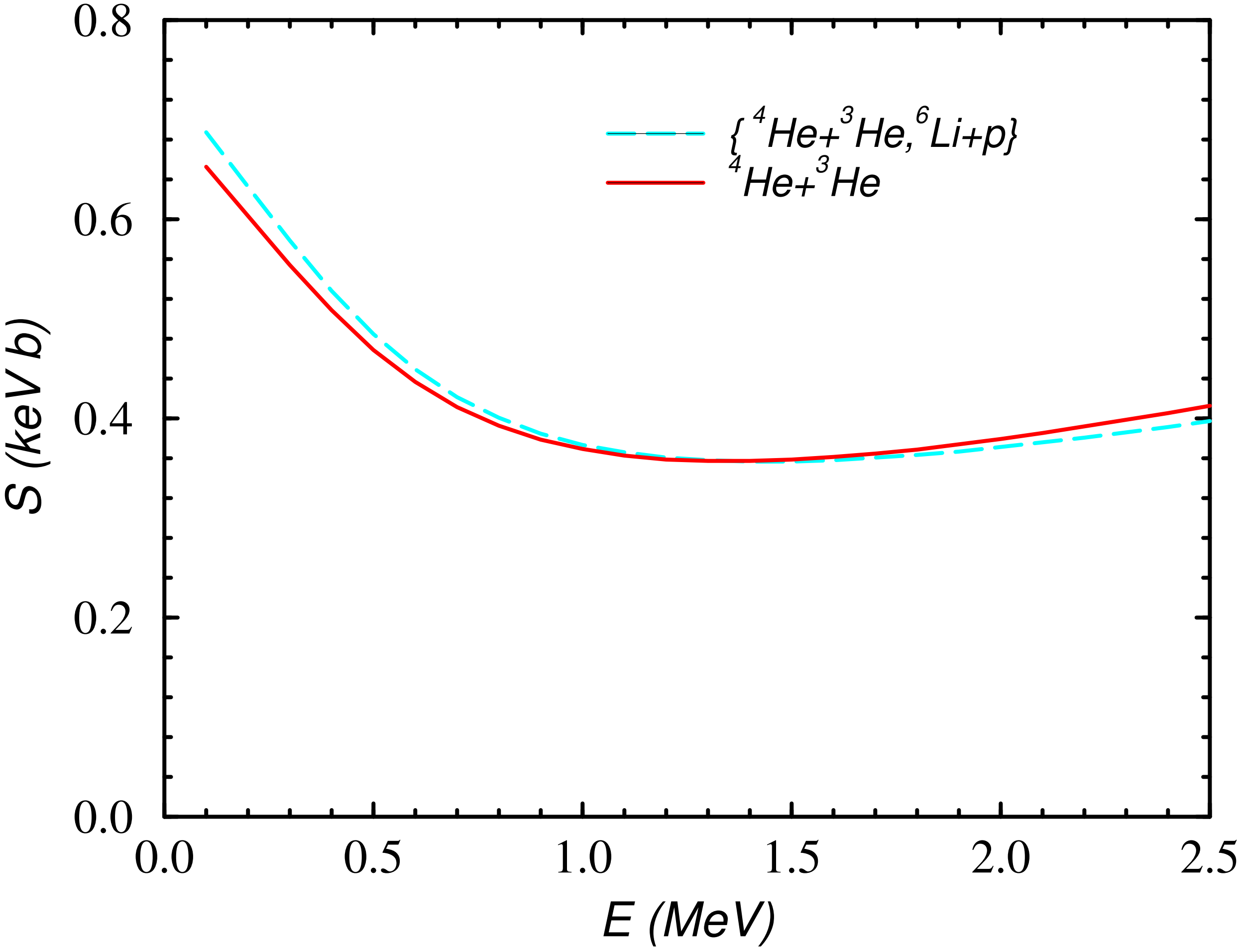


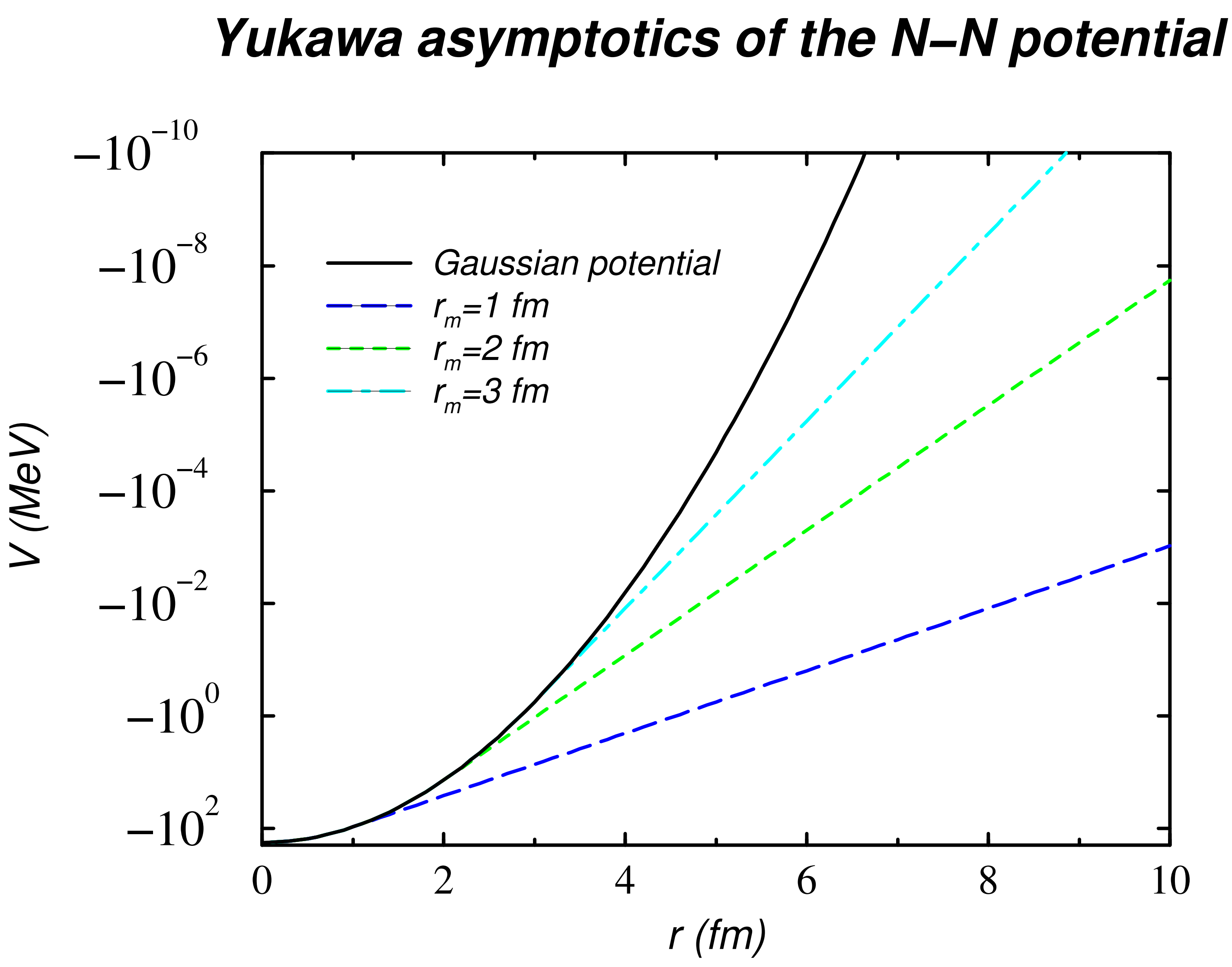
Asymptotic ^7Be wave function (no ^6Li+p)



Effect of $^{6}Li+p$ on $^{4}He(^{3}He,\gamma)^{7}Be$

!! Very preliminary !!




Yukawa asymptotics of the N–N potential
Gaussian potential
r_m=1 fm
r_m=2 fm
r_m=3 fm
V (MeV)
r (fm)
−10^−10
−10^−8
−10^−6
−10^−4
−10^−2
−10^0
−10^2
0
2
4
6
8
10

^{3}He(^{3}He, $2p$)^{4}He

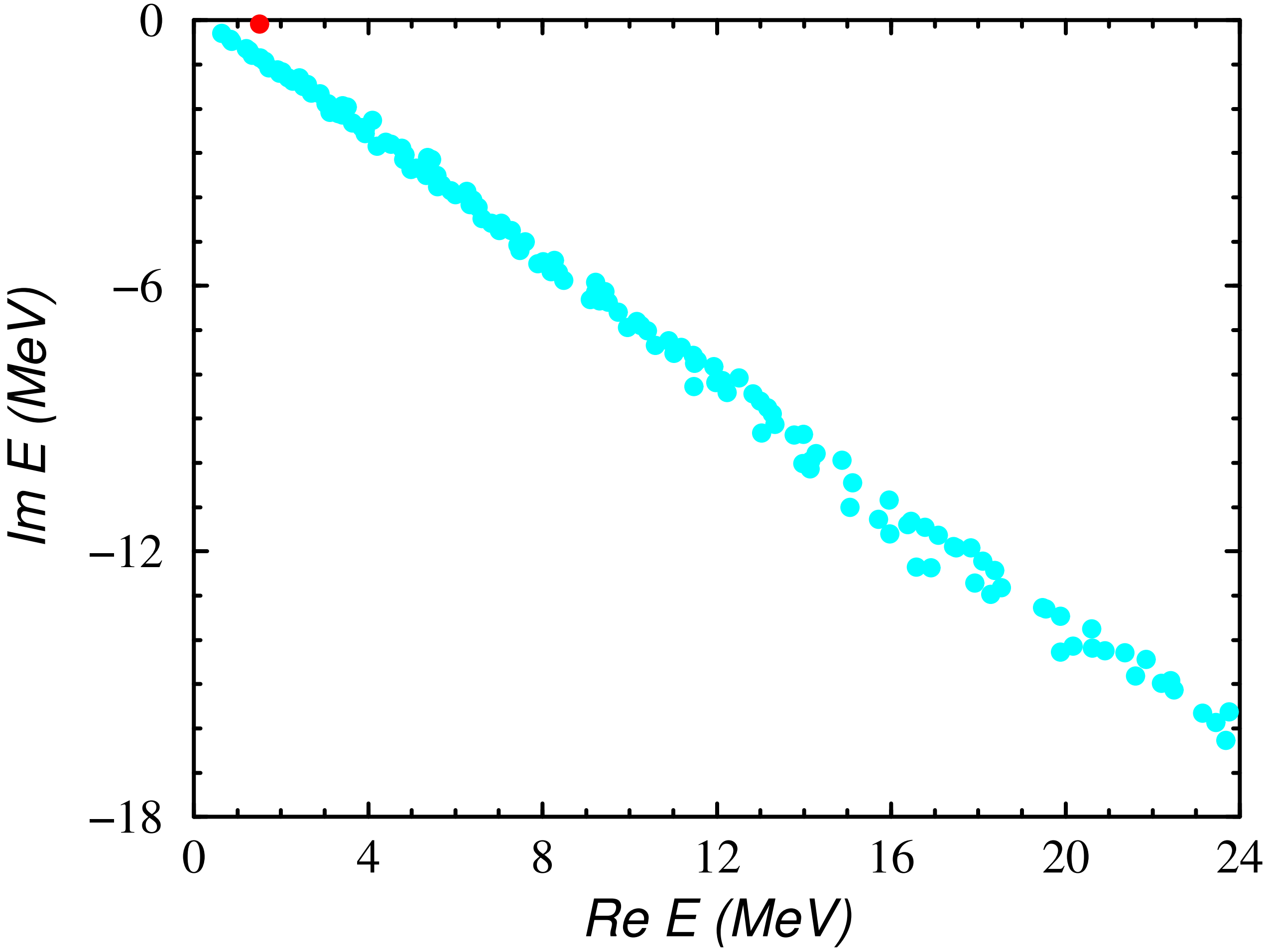
- Is there a low-energy resonance present which would change the balance of the neutrino-producing and non-neutrino-producing branches?
  - A resonance should come either from $^3\mathrm{He} + {}^3\mathrm{He}$ or $\alpha + p + p$: they can be studied separately
  - $^3\mathrm{He} + {}^3\mathrm{He}$, with the unpaired neutrons, is similar to $n + n$ $\Longrightarrow$ antibound state at negative energy? $\Longrightarrow$ exponentially increasing cross section not BW
  - In models with $0s$ ^{3}He the strong $^3S_1$ interaction is not present in the S-wave $^3\mathrm{He} + {}^3\mathrm{He}$ $\Longrightarrow$ the inclusion of the D-state inside ^{3}He would allow such components
- Precise model predictions for the energy dependence of the cross section are needed to clarify electron screening effects

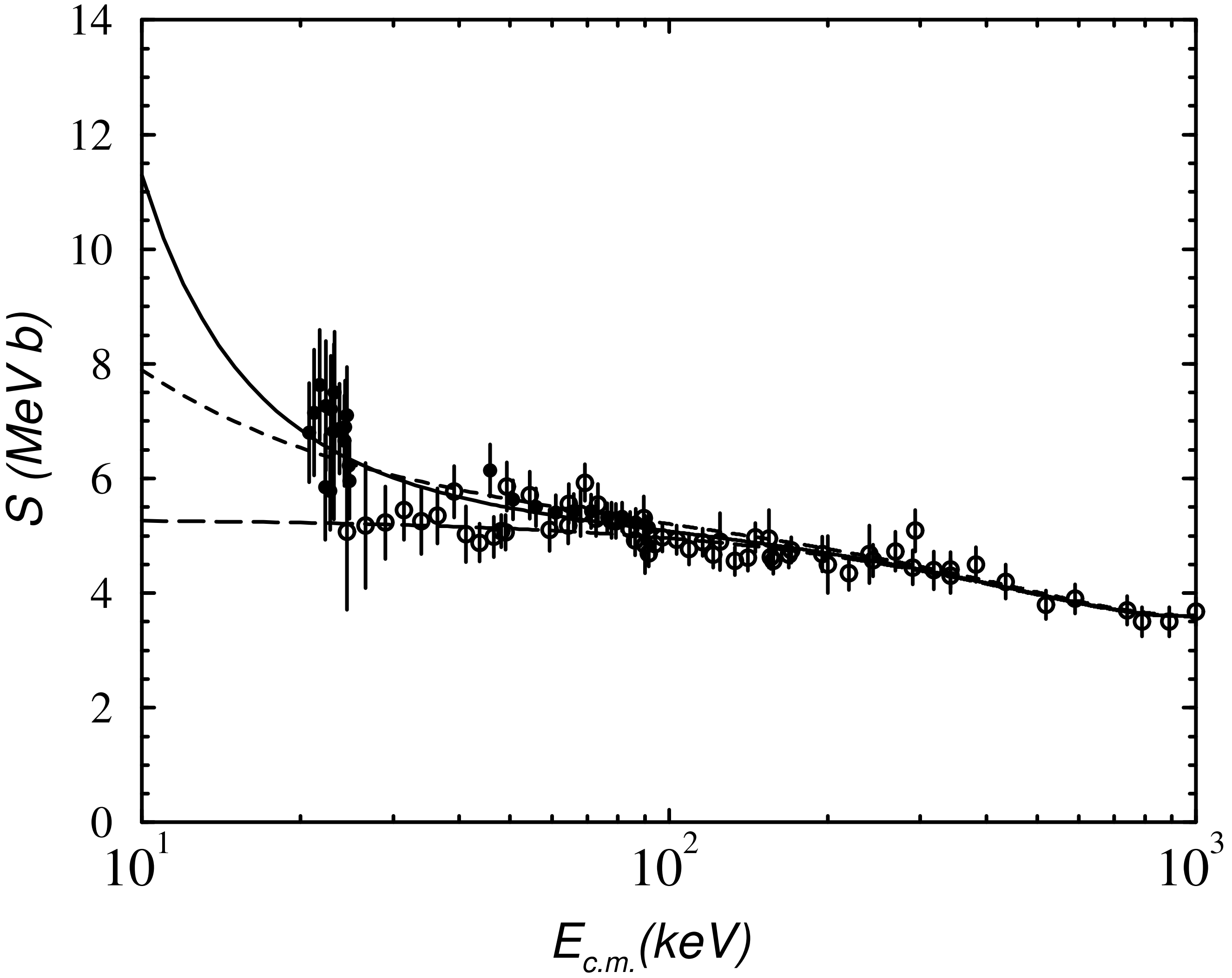
Continuum discretized coupled channel approximation to the reaction:

$$\begin{aligned}\Psi &= \psi\Big({}^3\mathrm{He} + {}^3\mathrm{He}\Big) \\ &+ \psi\Big(\alpha + (pp)_1\Big) + \psi\Big(\alpha + (pp)_2\Big) + \ldots \\ &+ \psi\Big(p + (\alpha p)_1\Big) + \psi\Big(p + (\alpha p)_2\Big) + \ldots\end{aligned}$$

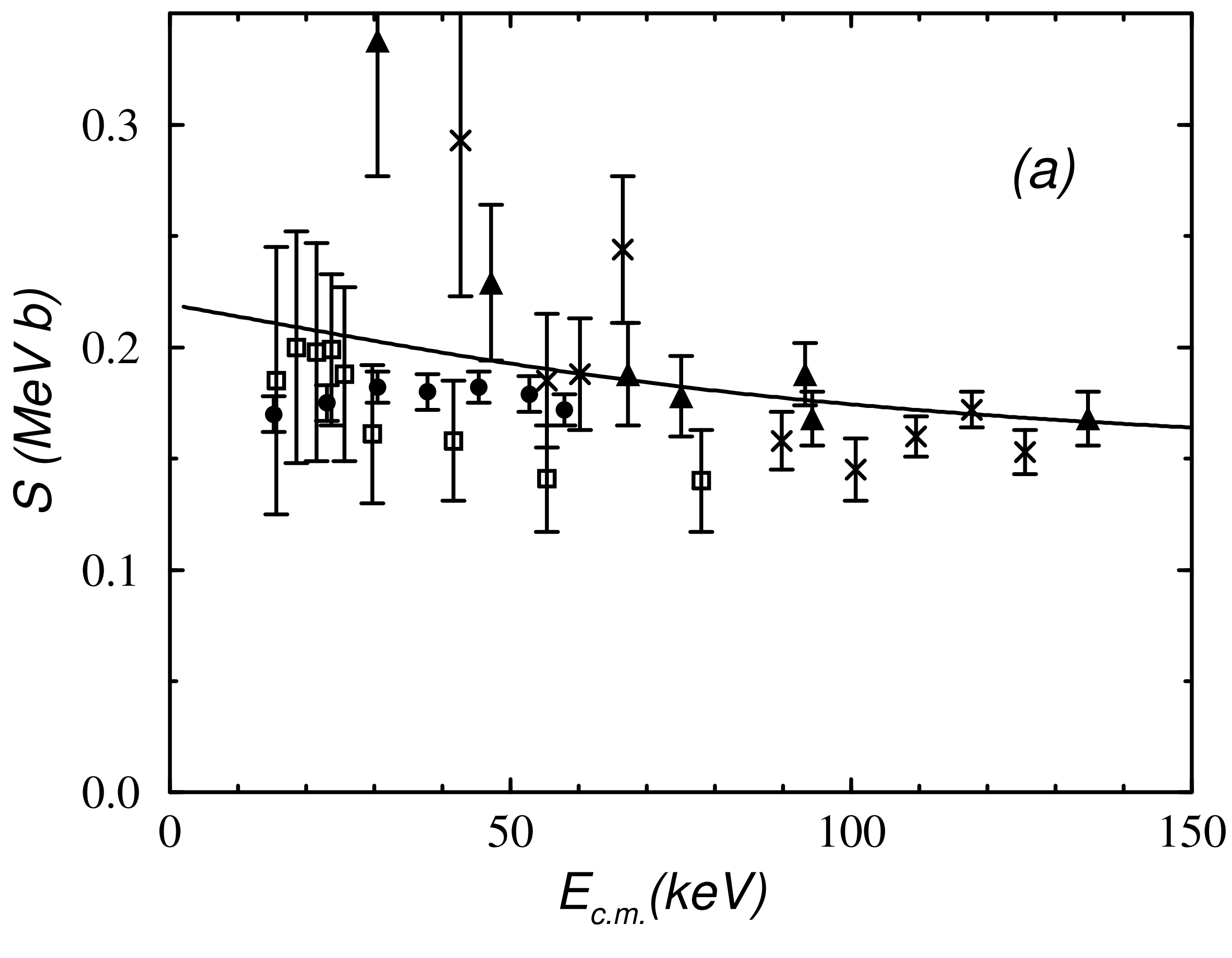
(pp) and (αp) are discretized continuum states of $p + p$ and ^{5}Li. The discretization is done by pseudobound states in a box, with the box size varying following a geometric progression.

Eigenvalues of H_θ for $J^\pi=0^+$ ^{6}Be states

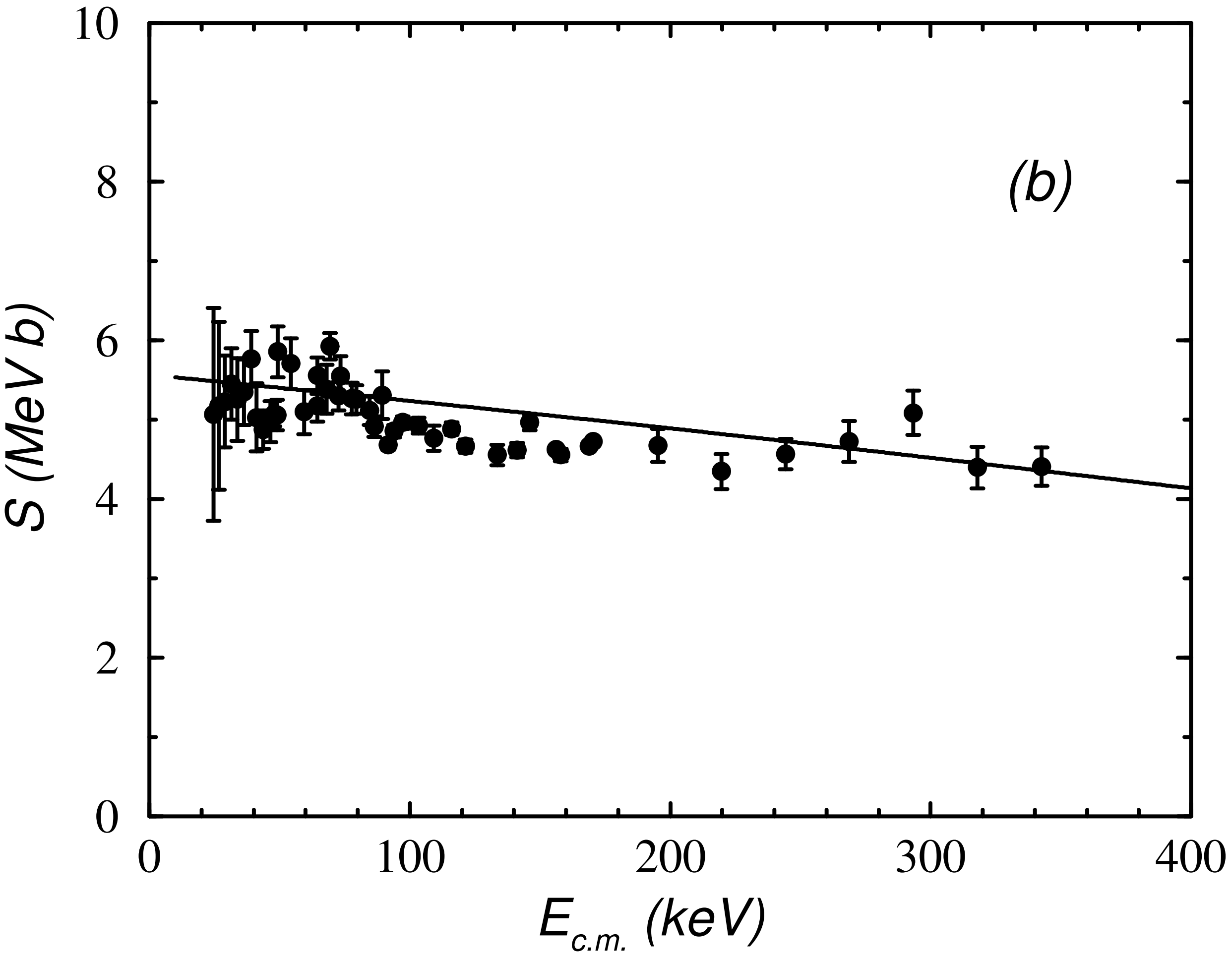




S (MeV b)
E c.m. (keV)
0
2
4
6
8
10
12
14
10^1
10^2
10^3



(a)
S (MeV b)
0.0
0.1
0.2
0.3
0
50
100
150
$E_{c.m.}(keV)$



(b)
S (MeV b)
E_{c.m.} (keV)
0
2
4
6
8
10
0
100
200
300
400

Summary (of the nuclear physics input)

- Experimental data are needed
  - Electron screening and stopping powers to understand $^3\mathrm{He}(^3\mathrm{He}, 2p)^4\mathrm{He}$ better
  - $^3\mathrm{He}(^3\mathrm{He}, 2p)^4\mathrm{He}$ to even lower energies
  - Quadrupole moment of $^7\mathrm{Be}$
  - Precise determination of $r(^7\mathrm{Be})$
  - High-statistics low-background $^8\mathrm{B}$ Coulomb dissociation experiments at several energies to map out the various electromagnetic multipole components
  - The "Ultimate Experiment": Radiative capture using $^7\mathrm{Be}$ beam in inverse kinematics
- Improved theoretical calculations are necessary
  - Realistic model calculation for $^3\mathrm{He}(^3\mathrm{He}, 2p)^4\mathrm{He}$ including the full three-body final state
  - Search for $^3\mathrm{He} + {}^3\mathrm{He}$ resonances in improved models
  - $^7\mathrm{Be}(p, \gamma)^8\mathrm{B}$ calculations in larger model spaces (eg. the effect of $^6\mathrm{Li} + p + p$?)
  - New and improved effective interactions for cluster models
  - The "Ultimate Calculation": Full solution of the 8-body problem with modern 2N and 3N interactions and EM currents (Argonne-Urbana-Los Alamos Quantum Monte Carlo group)

Summary (of the solar neutrino problem)

- Changing the nuclear cross sections cannot solve the problem (Heeger & Robertson): Let's assume $\phi(pp) = c_1\phi(pp)^{SSM}$, $\phi(^7\mathrm{Be}) = c_7\phi(^7\mathrm{Be})^{SSM}$, $\phi(^8\mathrm{B}) = c_8\phi(^8\mathrm{B})^{SSM}, \ldots$ with *arbitrary and physically unconstrained* constants c (eg. $c_7 = 0$ and $c_8 \neq 0$) and no luminosity constraint. All solar models are still ruled out at 95% CL.

- Neutrino oscillation is the favorite explanation

- **However** the neutrino oscillation is just a clever trick to introduce two free parameters with no effect on energy- and neutrino generation

- It is argued that $m_\nu \neq 0$ is natural because nothing forbids it (a counter example: gravitational vacuum energy density; seemingly nothing forbids it and still 56 orders of magnitude smaller than the natural scale $\longrightarrow$ a yet unknown symmetry forces it to be zero)

- "Free parameters" can be introduced without oscillation: eg. non-extensive Tsallis statistics instead of Maxwell: A 1% deviation from Maxwell can cause flux changes in the order of the SNP

- Another "free parameter" is still needed within the standard model, which can cause neutrino spectrum distortion