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NUCLEAR PHYSICS INPUT FOR SOLAR MODELS

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We discuss microscopic cluster model descriptions of two solar nuclear reactions, $^{7}\mathrm{Be}(p,\gamma)^{8}\mathrm{B}$ and $^{3}\mathrm{He}(^{3}\mathrm{He},2p)^{4}\mathrm{He}$. The low-energy reaction cross section of $^{7}\mathrm{Be}(p,\gamma)^{8}\mathrm{B}$, which determines the high-energy solar neutrino flux, is constrained by $^{7}\mathrm{Be}$ and $^{8}\mathrm{B}$ observables. Our results show that a small value of the zero-energy cross section is rather unlikely. In $^{3}\mathrm{He}(^{3}\mathrm{He},2p)^{4}\mathrm{He}$ we study the effects of a possible virtual state on the cross section. Although, we have found no indication for such a state so far, its existence cannot be ruled out yet. We calculate the $^{3}\mathrm{He}(^{3}\mathrm{He},2p)^{4}\mathrm{He}$ and $^{3}\mathrm{H}(^{3}\mathrm{H},2n)^{4}\mathrm{He}$ cross sections in a continuum-discretized coupled channel approximation, and find a good general agreement with the data.

1 Introduction

One of the most exciting fields of research these days is neutrino physics. Various experiments have been producing a large number of interesting results about the neutrino, yet after decades of work we still do not know even the most basic properties of these particles. The pioneering experiments that measure neutrinos coming from the sun produced the first, and still strongest, evidence for the possibility of nonzero neutrino mass, and hence for physics beyond the standard model. For a review of solar neutrino research, see [1].

Solar models contain input parameters from many fields of physics. For any reliable prediction of the solar neutrino fluxes, these parameters must be firmly established. Nuclear physics provides the rates of solar fusion reactions as input parameters for solar models. For a very recent review of our current understanding of these reactions, see [2]. In the following we shall discuss a microscopic model description of two important solar reactions, $^{7}\mathrm{Be}(p,\gamma)^{8}\mathrm{B}$ and $^{3}\mathrm{He}(^{3}\mathrm{He},2p)^{4}\mathrm{He}$, in detail.

2 Model and computational details

Currently the best *dynamical* description of $A = 6-8$ nuclei can be achieved by the microscopic cluster model. This model assumes that the nuclei consist of $2-3$ clusters. While the clusters are described by simple harmonic oscillator shell-model wave functions, the intercluster relative motions, which are the most important degrees of freedom, are treated rigorously. This model satisfies the correct bound- and scattering asymptotics of the wave functions, and satisfactorily reproduces the positions of the important breakup- and rearrangement channel thresholds and separation energies.

Because of the low temperature of our sun (on nuclear scales) the most effective energies for solar reactions are very low, being in the keV region. Thus, in order to calculate charged-particle reaction cross sections reliably, one must use computational methods which can supply correct bound- and scattering wave functions up to a few hundred fermi radii with high precision. Here we use the Kohn-Hulthén variational method for scattering states [3] and the Siegert variational method for bound states [4]. We briefly discuss these methods in a simplified manner, the generalization for realistic calculations is straightforward.

The Kohn-Hulthén method starts with the following trial wave function

$$\Psi^t = \sum_{i=1}^{N} c_i \varphi_i + \phi_E^- - S(E)\phi_E^+. \tag{1}$$

Here φ_i are square-integrable functions (Gaussians in our case) while ϕ_E^- and ϕ_E^+ are incoming and outgoing Coulomb functions with energy E, respectively. From the $\langle\, \delta\Psi^t \mid \widehat{H} - E \mid \Psi^t \,\rangle = 0$ projection equation one gets a set of linear equations that can be solved for the c_i coefficients and the S scattering matrix. For many-body systems one can use basis functions that are made by matching φ_i with the Coulomb functions in the external regions. This way all many-body matrix elements can be reduced to analytic forms plus one-dimensional integrals [3].

The Siegert variational method uses a trial function with purely outgoing asymptotics:

$$\Psi^t = \sum_{i=1}^{N} c_i \varphi_i + c_{N+1}\phi^+. \tag{2}$$

After the variation one arrives at a set of linear equations which is underdetermined (with $N+2$ unknowns: $c_1, c_2, \ldots, c_{N+1}, E$). It has solutions, the bound states, only at discrete energies, where the determinant of the system of equations is zero. A matching technique similar to the scattering case can be used to calculate all matrix elements analytically. Using the resulting wave functions the reaction cross sections can be calculated. In order to get rid of the trivial exponential energy dependence of the cross sections, which comes from the Coulomb barrier penetration, we use the astrophysical S-factor

$$S(E) = \sigma(E) E \exp\Big[2\pi\eta(E)\Big], \qquad \eta(E) = \frac{\mu Z_1 Z_2 e^2}{k\hbar^2}. \tag{3}$$

The model and methods described above have been used to study the $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ and $^3\mathrm{He}(^3\mathrm{He},2p)^4\mathrm{He}$ solar reactions in $^4\mathrm{He}+{}^3\mathrm{He}+p$ and $\{^3\mathrm{He}+{}^3\mathrm{He},{}^4\mathrm{He}+p+p\}$ cluster models, respectively.

3 $^7\mathrm{Be}(\mathrm{p},\gamma)^8\mathrm{B}$ cross section constrained by A=7 and 8 observables

The most uncertain nuclear input parameter in standard solar models is the low-energy $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ radiative capture cross section. This reaction produces $^8\mathrm{B}$ in the sun, whose β^+ decay is the main source of the high-energy solar neutrinos. Many present and future solar neutrino detectors are sensitive mainly or exclusively to the $^8\mathrm{B}$ neutrinos. The predicted $^8\mathrm{B}$ neutrino flux is linearly proportional to S_{17}, the $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ astrophysical S factor at solar energies ($E_{cm}=20$ keV). Thus, the value of $S_{17}(20\text{ keV})$ is a crucial input parameter in solar models. The six radiative capture measurements performed to date give, after extrapolations from higher energies, $S_{17}(0)$ between 15 eVb and 40 eVb, with a weighted average of 22.4 ± 2.1 eVb [5], while a recent Coulomb dissociation measurement gives $S_{17}(0)=16.7\pm3.2$ eVb [6]. The theoretical predictions for $S_{17}(0)$ also have a huge uncertainty, as the various models give values between 16 eVb and 30 eVb.

The low solar energies mean that the reaction takes place well below the Coulomb barrier. In such cases the radiative capture cross section gets contributions almost exclusively from the external nuclear regions ($r>6-8$ fm). At such distances the scattering- ($^7\mathrm{Be}+p$) and bound state ($^8\mathrm{B}$) wave functions are fully determined, provided the scattering phase shifts and bound state asymptotic normalizations are known [7]. At solar energies the phase shifts coincide with the (almost zero) hard sphere phase shifts, while the bound state wave function in the external region behaves like $\bar{c}W^+(kr)/r$, where W^+ is the Whittaker function and $\bar{c}$ is the asymptotic normalization. So the only unknown parameter that governs the $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ reaction at low energies is the $\bar{c}$ value. The $\bar{c}$ normalization depends mainly on the effective $^7\mathrm{Be}$–p interaction radius. A larger radius results in a lower Coulomb barrier, which leads to a higher tunneling probability into the external region, and hence to a higher cross section. We believe that the best way to constrain the potential radius is to study some key properties of the $A=7$ and 8 nuclei [8]. The observables that are most sensitive to the interaction radius are “size” type properties, for example, quadrupole moment, radius, Coulomb displacement energy [9], etc.

We use an eight-body three-cluster model which is variationally converged and virtually complete in the $^4\mathrm{He}+{}^3\mathrm{He}+p$ cluster model space [10]. We find that the low-energy astrophysical S factor is linearly correlated with the quadrupole moment of $^7\mathrm{Be}$ (Fig. 1). This quantity, Q_7, has not been measured yet, but

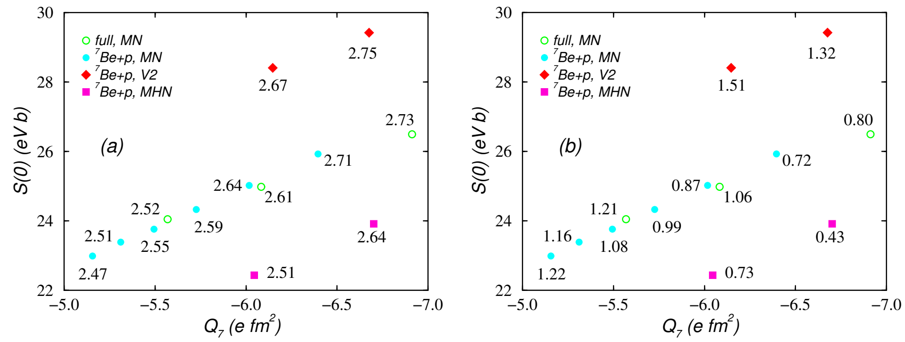

Figure 1: Correlation between the zero-energy astrophysical S factor of the $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ reaction and the quadrupole moment of ^{7}Be in our microscopic eight-body model. The results of several calculations, using various $N-N$ interactions and model spaces, are shown. Within one model space and interaction, the different results come from different cluster sizes. The numbers in Fig. (a) are the calculated ^{8}B point-nucleon radii (in fm), while in Fig. (b) they are the $r^2(^8\mathrm{B}) - r^2(^7\mathrm{Be})$ values (in fm^2) for the various models. The phenomenological values are $r(^8\mathrm{B}) = 2.50 \pm 0.04$ fm and $r^2(^8\mathrm{B}) - r^2(^7\mathrm{Be}) \approx 0.9$ fm^2.

the model itself predicts it to be between -6 e fm^2 and -7 e fm^2. In addition to the Q_7 dependence, there is a sensitivity of $S_{17}(0)$ on the employed $N-N$ interaction, as shown in Fig. 1. We found that the MN interaction is the most self-consistent in describing the $A=7$ and 8 nuclei and the $N+N$ systems. Thus, our model predicts $S_{17}(0) = 25 - 26.5$ eVb. We mention, however, that the construction and use of other high-quality effective $N-N$ interactions would be desirable in order to check our findings.

We have also tested how our model reproduces other "size" observables [11]. The quantity that is most sensitive to the effective $^7\mathrm{Be}-p$ interaction radius is $r^2(^8\mathrm{B}) - r^2(^7\mathrm{Be})$. However, a precise experimental determination of this quantity is very difficult. Originally the ^{7}Be and ^{8}B radii were determined from interaction cross sections by using Glauber-type models with uniform density distributions for the nuclei [12]. Recently a more precise ^{8}B radius was extracted from interaction cross section data by taking into account the $^7\mathrm{Be}+p$ nature of ^{8}B [13]. The resulting point-nucleon radius is $r(^8\mathrm{B}) = 2.50 \pm 0.04$ fm, and hence $r^2(^8\mathrm{B}) - r^2(^7\mathrm{Be}) \approx 0.9$ fm^2. The model still uses the Glauber result for the ^{7}Be radius. A few-body $^4\mathrm{He} + {}^3\mathrm{He}$ description of ^{7}Be would probably slightly increase $r(^7\mathrm{Be})$ and consequently $r(^8\mathrm{B})$.

In Fig. 1 we give the ^{8}B radii and $r^2(^8\mathrm{B}) - r^2(^7\mathrm{Be})$, calculated for the various model spaces, interactions, etc. It appears from Fig. 1(a) that the phenomenological $r(^8\mathrm{B})$ value would suggest a $\approx$10% reduction in $S_{17}(0)$ rel-

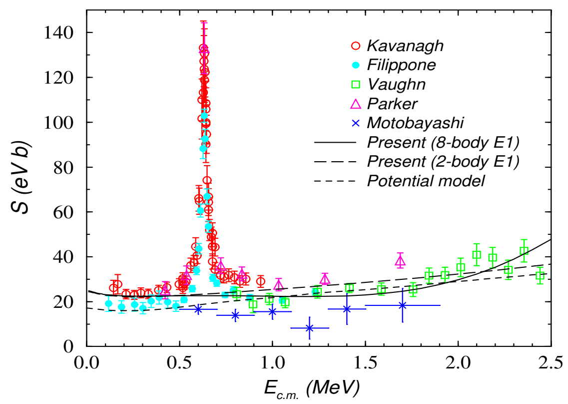

Figure 2: Astrophysical S factor for the $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ reaction in our eight-body model. The symbols show the various experimental data (see [14] for references). The solid and long-dashed lines are the $E1$ components of the S factors in our model with and without antisymmetrization in the electromagnetic transition matrix, respectively. The short-dashed line is the result of a typical potential model [7].

ative to our MN prediction. However, it is interesting to note that increasing the model space (open circles) reduces the calculated $r(^8\mathrm{B})$ at a given Q_7. If further model-space extensions resulted in the same behavior, then once again our MN results would be the most self-consistent. Fig. 1(b) shows that $r^2(^8\mathrm{B}) - r^2(^7\mathrm{Be})$ seems to be already too small for both the MN and MHN interactions. Model-space extensions bring the MN results toward the phenomenological value. The $^8\mathrm{Li}-^8\mathrm{B}$ Coulomb displacement energy shows similar behavior to the observables in Fig. 1 [11].

While the zero-energy cross section is sensitive only to the asymptotic parts of the wave functions, with increasing energy the internal wave functions become more and more important. The internal wave functions are sensitive to effects like the exchange of the incoming proton with a proton in $^7\mathrm{Be}$, the excitation of the $^7\mathrm{Be}$ core by the incoming proton, $^7\mathrm{Be}$ deformations, etc. The influence of these off-shell effects on $S(E)$ was studied in our eight-body model [14]. We took the $^7\mathrm{Be}-p$ relative motion wave functions coming from the cluster model, and used them as if they came from a potential model. This way the antisymmetrization, which is the biggest off-shell effect, was neglected in the electromagnetic transition matrix. One can see in Fig. 2 that the resulting S factor (long dashed line) has quite different energy dependence than the full microscopic S factor (solid line). We also show an S factor coming from a potential model (short dashed line). This calculation demonstrates that there are strong off-shell effects present in $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$, which have to be taken into account for any reliable extrapolation of high-energy measurements.

All existing microscopic calculations for $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ use effective $N-N$ interactions with Gaussian shape. One can argue that using potentials with the correct Yukawa asymptotics would change $S(0)$. We studied this problem by using the MN interaction with Gaussian shape matched with a Yukawa tail [11]. We found that in the perturbative regime, where such a study makes sense, $S(0)$ is insensitive to the Yukawa tail, although the $A=7$ and 8 binding energies change significantly, and the overall strengths of the potentials have to be refitted.

Another interesting question is the effect of further extensions in the model space by including, for example, $^6\mathrm{Li}+p+p$ and other configurations in $^8\mathrm{B}$. As an exploratory investigation, we studied the effects of $^6\mathrm{Li}+N$ on $^7\mathrm{Li}$ and $^7\mathrm{Be}$[11]. An interesting result is that while $^7\mathrm{Li}$ is not affected by the $^6\mathrm{Li}+n$ channel, some $^7\mathrm{Be}$ properties, like the quadrupole moment, is strongly influenced by $^6\mathrm{Li}+p$. We observe a slight change also in the energy dependence of the $^4\mathrm{He}(^3\mathrm{He},\gamma)^7\mathrm{Be}$ S factor if $^6\mathrm{Li}+p$ is included. Further studies are in progress.

4 The ^{3}He(^{3}He,2p)^{4}He reaction

The $^3\mathrm{He}(^3\mathrm{He},2p)^4\mathrm{He}$ reaction competes with the $^7\mathrm{Be}$ producing branch of the solar p-p chain [1]. Thus, it indirectly affects the $^7\mathrm{Be}$ and $^8\mathrm{B}$ neutrino fluxes. There are two interesting problems related to this reaction: i) a possible low-energy resonance in the cross section would suppress the high-energy solar neutrino fluxes [15]; ii) this is the only solar reaction whose cross section has been measured down to solar energies, and the effect of electron screening is still not fully understood [16]. We studied these problems in a six-body $\{^3\mathrm{He}+{}^3\mathrm{He}, {}^4\mathrm{He}+p+p\}$ cluster model [17].

If there is a resonance in the $^3\mathrm{He}(^3\mathrm{He},2p)^4\mathrm{He}$ reaction cross section, then it comes from either $^3\mathrm{He}+{}^3\mathrm{He}$ or $^4\mathrm{He}+p+p$. Interestingly, the second case is easier to study despite its three-body nature. We searched for high-lying narrow resonances in $^4\mathrm{He}+p+p$ using the complex scaling method that can handle the three-body Coulomb asymptotics correctly [18,17]. We found no such states. The $^3\mathrm{He}+{}^3\mathrm{He}$ channel is more difficult and more interesting. We mention here only one interesting feature. The $^3\mathrm{He}+{}^3\mathrm{He}$ system is similar in many respects to the $n+n$ system, which has a virtual 0^+ state with negative energy and negative imaginary wave number. If there is a virtual state present in $^3\mathrm{He}+{}^3\mathrm{He}$ then it would result in a $^3\mathrm{He}(^3\mathrm{He},2p)^4\mathrm{He}$ cross section that is singular at the (unphysical) negative $^3\mathrm{He}+{}^3\mathrm{He}$ pole energy, and has a $1/E$ energy dependence at low positive energies. We show the effect

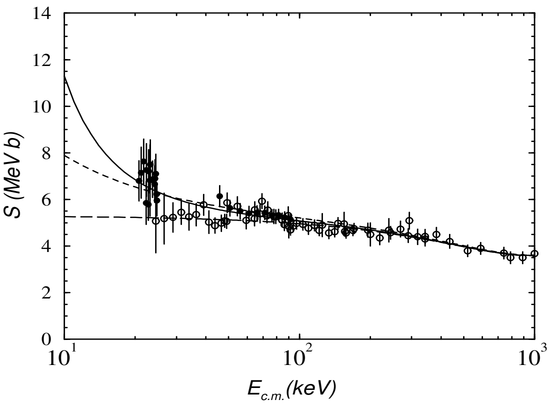

Figure 3: Astrophysical S factor for the $^3\mathrm{He}(^3\mathrm{He},2p)^4\mathrm{He}$ reaction. The data points are from [16] and references therein. The solid line is the fitted raw cross section, while the long dashed line is the bare cross section, determined by removing the electron screening effect with $U = 323$ eV screening potential. The short dashed line shows the effect of a hypothetical zero-energy virtual state on the bare cross section. The strength of the virtual state is artificially set to 5 keV.

of such a hypothetical state in Fig. 3. The observed rise in the measured cross section (relative to the bare cross section) is attributed to the effect of electron screening. One can see that a virtual state could mimic a similar behavior. We have searched for virtual states in $^3\mathrm{He} + {}^3\mathrm{He}$ in a simple cluster model, and found none so far. Further studies in a more realistic model are in progress [17].

The understanding of the energy dependence of the cross section is also important for the study of electron screening effects. The screening potential, extracted in [16], seems to be larger than predicted by theory. We studied the $^3\mathrm{He}(^3\mathrm{He},2p)^4\mathrm{He}$ and the mirror $^3\mathrm{H}(^3\mathrm{H},2n)^4\mathrm{He}$ reactions in large-space cluster models in the continuum discretized coupled channel approximation [17]. The results are in Fig. 5. We observe a good general agreement with the data in both the absolute normalization and shape. However, a marked disagreement exists at very low energies in $^3\mathrm{H}(^3\mathrm{H},2n)^4\mathrm{He}$ between our result and the very precise Los Alamos data. Further investigation to understand this discrepancy is in progress.

5 Conclusion

We have studied the $^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$, and $^3\mathrm{He}(^3\mathrm{He},2p)^4\mathrm{He}$ solar nuclear reactions in microscopic cluster models.

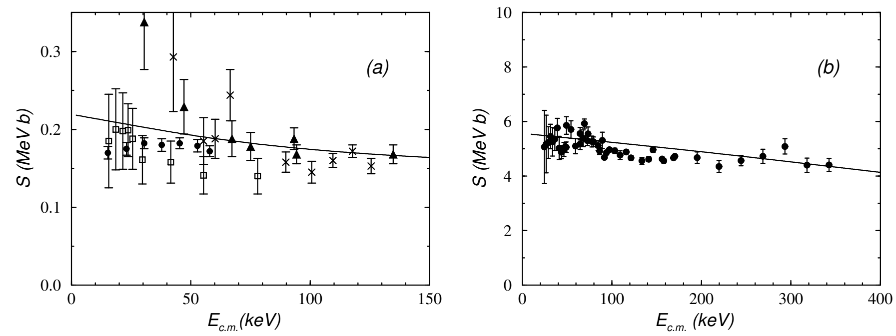

Figure 4: Astrophysical S factor for the (a) $^3\mathrm{H}(^3\mathrm{H},2n)^4\mathrm{He}$ and (b) $^3\mathrm{He}(^3\mathrm{He},2p)^4\mathrm{He}$ reactions. In Fig. (b) only the data of Krauss are shown (cf. [16]). The curves come from our six-body calculations in a continuum discretized coupled channel approximation.

Our $S_{17}(0) = 25 - 26.5$ eVb astrophysical S factor is slightly higher than, but consistent with the value[5] 22.4 ± 2.1 eVb currently used in standard solar models. Our results show that certain ^{7}Be and ^{8}B observables, like radius and quadrupole moment, can establish a region of the possible values of $S_{17}(0)$. Our model shows that a small value of $S_{17}(0)$ is rather unlikely, unless some very important ingredient is missing in our approach. Currently we have no candidate for such a missing element. We would like to emphasize the need for further experiments by using both the radiative capture technique[19] and Coulomb dissociation[20]. A measurement of the ^{7}Be quadrupole moment and a precise extraction of the ^{7}Be radius would also be very beneficial.

Our calculations for $^3\mathrm{He}(^3\mathrm{He},2p)^4\mathrm{He}$ show no resonances or virtual states either in the $^3\mathrm{He}+{}^3\mathrm{He}$ or in the $^4\mathrm{He}+p+p$ channels. Further studies of $^3\mathrm{He}+{}^3\mathrm{He}$ in more realistic models are necessary. We calculated the $^3\mathrm{He}(^3\mathrm{He},2p)^4\mathrm{He}$ and $^3\mathrm{H}(^3\mathrm{H},2n)^4\mathrm{He}$ cross sections and found a good general agreement with existing data.

Solar model independent analyses show that the measured solar neutrino rates cannot be reproduced by arbitrarily changing the normalizations of the neutrino spectra[21] (the shapes of the spectra are currently beyond any doubt). Thus, the solar fusion rates themselves, which can change only the absolute normalizations of the fluxes, cannot solve the solar neutrino problem. In fact we have more efficient ways within the standard model to change the absolute normalizations, than nuclear physics. For instance, by slightly deviating from the Maxwell-Boltzmann thermal statistics one can cause as large changes in the various neutrino fluxes as those that constitute the solar neutrino problem itself[22]. However, in order to have a solution of the solar neutrino problem within the standard model, we would still need a yet unknown mechanism that could cause spectrum distortion, like the MSW neutrino oscillation in a non-standard model[1].

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