On the three-body continuum spectrum of ^{6}He

Attila Csótó
Department of Atomic Physics, Eötvös University, Puskin utca 5-7, H–1088 Budapest, Hungary

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In recent publications Cobis, Fedorov, and Jensen claim the existence of several previously unknown low-lying narrow resonances in ^{6}He. I show that the distribution of the S-matrix poles corresponding to these states is unphysical. This casts doubt on the results of those works concerning resonances.

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Recently a series of papers has been published on three-body continuum calculations for the neutron-halo nuclei ^{6}He and ^{11}Li [1]. The authors report on numerous previously unknown states they find in these nuclei at low energies. Most importantly they claim to have found the much debated soft dipole resonances in ^{6}He and ^{11}Li. I argue that the presentation of Ref. [1] is rather misleading, because important information about the calculations, that would have cast doubt on the results, were not mentioned. As I show, the results regarding the existence of low-energy narrow states in ^{6}He are highly questionable.

Three-body resonances in an A=6 nucleus, ^{6}Li, were first studied in Ref. [2] in an $\alpha+p+n$ model. A systematic search for such states in ^{6}He, ^{6}Li, and ^{6}Be was first performed in Ref. [3] with the aim to confirm or refute the existence of the soft dipole (1^-) resonance in ^{6}He. Only the known states were found in the three nuclei, and no evidence for the existence of the soft dipole resonance in ^{6}He surfaced. As was stated in Ref. [3], the method was numerically not stable enough for broad states ($\Gamma \gg E$), so the possible existence of such states cannot be ruled out based on that work.

The ^{6}He nucleus was studied also in an $\alpha + n + n$ model with structureless α in Refs. [4,5] using different interactions and different methods. They both find several previously unknown states. While there is a good agreement between Refs. [4,5] in the ordering and spin-parities of the new states, the Ref. [4] resonances are relatively narrow, whereas all new states in Ref. [5] are very broad. The relatively narrow states found in Ref. [4] should have been seen in Ref. [3], as the method used there was adequate for them. However, neither the original work [3] nor new calculations [6] show such states. On the other hand, the Ref. [3] model cannot rule out the broad states of Ref. [5]. We note that it is quite possible that despite the big differences in the widths, the Ref. [4] and Ref. [5] states correspond to each other, and the differences come mainly from the different Hamiltonians. Test calculations using the same Hamiltonian would be desirable.

Finally, the latest experiments do not seem to support the existence of any new narrow states in ^{6}He [7].

In contrast to all previous works which suggested only a few (if any) new resonances in ^{6}He, Ref. [1] predicts several rather narrow states in each J^π channel. The authors of Ref. [1] avoid the use of the words "state" or "resonance" in connection with the S-matrix poles they find. However, one must realize that according to mathematical theorems, for well-behaved potentials all poles of an S matrix in the meromorphic region of the potential are physical, and correspond to resonances [8]. Thus, if the mathematical conditions for the potentials are satisfied, the analytic continuation of the S matrix is done properly, and the whole procedure is numerically stable, then all the poles in Ref. [1] should correspond to real resonances of ^{6}He.

In Fig. 1 the complex-energy positions of the first four S-matrix poles found [9] by the authors of Ref. [1] in the $J^\pi = 1^-$ Hamiltonian are shown. Although all these poles (and possibly more) were known to the authors, they elected to show only the first two in each partial waves in Ref. [1]. The distribution of the poles in Fig. 1 is *clearly unphysical*, and thus some or all of them must be artifacts. The origin of these spurious states can be threefold: i) the "effective potentials" appearing in the method of [1] do not satisfy the necessary mathematical conditions; ii) the analytic continuation is not done with sufficient care in [1]; iii) the whole method of Ref. [1] is questionable. Personally I think that point ii) is probably the (main) source of the problem. The three-body problem has a rather complicated analytic structure at complex energies especially if there are resonances in the two-body subsystems, like in $\alpha + n + n$. The analytic structure of the Riemann energy surface was discussed in detail, e.g., in Ref. [10]. One can see in Ref. [10] that, for example, the resonant poles of the two-body subsystems appear as complex-energy thresholds with two-body branch cuts in the complex plane in the three-body problem (for a numerical illustration, see also [11]). It means that for $\alpha + n + n$ there are branch cuts starting at the $\alpha + n$ pole energies $(0.77 - i0.32)$ MeV and $(1.97 - i2.61)$ MeV of Ref. [1,9], respectively for $J^\pi = 3/2^-$ and $1/2^-$. It seems rather plausible that the two highest-energy points in Fig. 1 are part of the $3/2^-$ cut. If this is so, then further "poles" should be found lying on this line closer to the imaginary E axis.

In order to be able to understand the nature of the first two poles, one should know more about the details of the analytic continuation used in Ref. [1]. We mention just one example that the authors used rather unorthodox conventions: they defined the three-body branch cut along the *negative* real energy axis, thus mapping the left and right half k-planes onto a Riemann surface instead of the top and bottom half-planes. This may seem just a matter of choice, but it might violate some fundamental symmetries as well.

In Ref. [1] it is shown that the asymptotic part of the wave function for the 1^- state starts at very large r. This can imply that other methods might not be able to handle this behavior correctly, and as a consequence might miss the 1^- state. This is a valid argument, so we checked it in the complex scaling method used in Ref. [3]. The 1^- state did not appear even if the range of the basis extended beyond 100 fm. At the same time the position of the 2^+ state remained remarkably stable despite the fact that such a basis is numerically unfavorable.

An implicit argument in Ref. [1,9], to support the existence of the low-lying states in ^{6}He, is the attractive nature of both the $n+n$ and $\alpha+n$ forces (in other words, the large scattering lengths) in the crucial partial waves. Although this may seem to be a logical argument, there is at least one well-known counter-example to it: the nonexistence of the $1/2^+$ state of the three-neutron system [12,13]. The most important configuration of such a state would be an $L=0$ relative motion between a $^1S_0$ dineutron and the third neutron. This would contain the attractive $^1S_0$ $N-N$ interaction in all two-body subsystems. Yet there is no evidence, either theoretical or experimental, for the existence of such a state. I think that a good test of the methods of Ref. [1] would be the $3n$ system.

Finally, I would like to emphasize that a future measurement of some real-energy observables, like the dipole strength function, cannot be used as a proof for the existence of the states in Ref. [1], even if the data happened to agree with the theoretical prediction. As it was shown in Ref. [3,11], some structures of the three-body continuum can come not only from three-body resonances but, for example, from the two-body substructure. The sequential breakup of ^{6}He might produce a strength function similar to those in Ref. [1] without any 1^- three-body resonance.

In conclusion, I have pointed out that the distribution of the complex S-matrix poles in Ref. [1,9] is clearly unphysical. It seems that most of the complex-energy results of Ref. [1] might be artifacts caused by the unsatisfactory handling of the analytic continuation. I think it would be very beneficial if the authors of Ref. [1] tried to compare their results to other published ones, e.g. to Ref. [5], using the same potentials (as it should have been done at least for tests in the original work).

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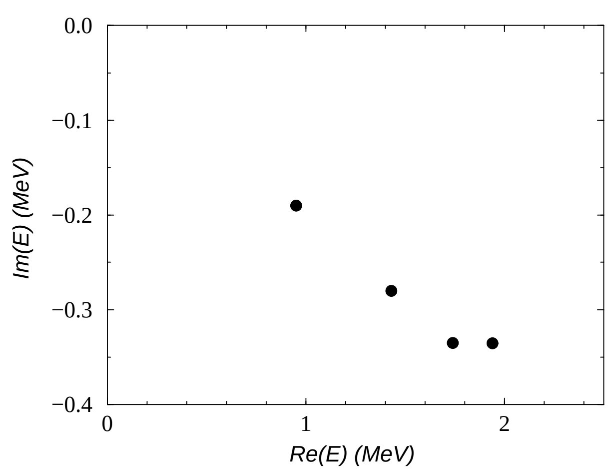

FIG. 1. Positions of the the first four poles of the $J^\pi = 1^-$ S matrix of Ref. [1,9] on the complex-energy plane.