

# FINE-TUNING CARBON-BASED LIFE IN THE UNIVERSE BY THE TRIPLE-ALPHA PROCESS IN RED GIANTS

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Through the triple-alpha process occurring in red giant stars the bulk of the carbon existing in our universe is produced. We calculated the change of the triple-alpha reaction rate for slight variations of the nucleon-nucleon force using a microscopic 12-body model. Stellar model calculations for a low-mass, intermediate-mass and massive star using the different triple-alpha reaction rates obtained with different strengths of the N-N interaction have been performed. Even with a change of 0.4% in the strength of N-N force, carbon-based life appears to be impossible, since all the stars then would produce either almost solely carbon or oxygen, but could not produce both elements.

## 1 Introduction

The Anthropic Principle in its weak form can be formulated in the following way: *The observed values of all physical and cosmological quantities are not equally probable but they take on values restricted by the requirement that there exist sites where carbon-based life can evolve and by the requirement that the universe be old enough for it to have already done so*<sup>1</sup>. The triple-alpha process occurring in the helium burning of red giants is of special significance with respect to the anthropic principle. Through this process the bulk of carbon, e.g., the carbon isotope  $^{12}\text{C}$ , existing in our universe is produced. Carbon is synthesized further by alpha capture to oxygen, e.g., to the oxygen isotope  $^{16}\text{O}$ , leading to abundance ratio of  $^{12}\text{C} : ^{16}\text{O} \approx 1 : 2$ . Without the production of an appreciable amount of carbon, obviously no carbon-based life could have developed in the universe. On the other hand, the production of oxygen is also absolutely necessary, because no spontaneous development of carbon-based life

is possible without the existence of water ( $\text{H}_2\text{O}$ ).

The reason for the relevance of the triple-alpha process with respect to the anthropic principle lies also in the fact that one has to deal with physical quantities that are in the realm of experimentally verifiable and theoretically calculable physics. This is for instance hardly the case for the much less well-known and complicated science necessary for the description of the Big Bang as well as for the creation and evolution of life on earth.

The formation of  $^{12}\text{C}$  in hydrogen burning is blocked by the absence of stable elements at mass numbers  $A = 5$  and  $A = 8$ . Öpik and Salpeter<sup>2,3</sup> pointed out that the lifetime of  $^8\text{Be}$  is long enough, so that the  $\alpha + \alpha \rightleftharpoons ^8\text{Be}$  reaction can produce macroscopic amounts of equilibrium  $^8\text{Be}$  in stars. Then, the unstable  $^8\text{Be}$  could capture an additional  $\alpha$ -particle to produce stable  $^{12}\text{C}$ . However, this so-called triple-alpha reaction has very low rate since the density of  $^8\text{Be}$  in the stellar plasma is very low, because of its short lifetime of  $10^{-16}$  s.

Hoyle<sup>4</sup> argued that in order to explain the measured abundance of carbon in the Universe, the triple-alpha reaction cannot produce enough carbon in a non-resonant way, therefore it must proceed through a hypothetical resonance of  $^{12}\text{C}$ , thus strongly enhancing the cross section. Hoyle suggested that this resonance is a  $J^\pi = 0^+$  state at about  $\varepsilon = 0.4$  MeV (throughout this paper  $\varepsilon$  denotes resonance energy in the center-of-mass frame relative to the three-alpha threshold, while  $\Gamma$  denotes the full width). Subsequent experiments indeed found a  $0^+$  resonance in  $^{12}\text{C}$  in the predicted energy region by studies of the reaction  $^{14}\text{N}(\text{d},\alpha)^{12}\text{C}$  and the  $\beta^-$ -decay of  $^{12}\text{B}$ <sup>4,5</sup>. It is the second  $0^+$  state ( $0_2^+$ ) in  $^{12}\text{C}$ . Its modern parameters<sup>6</sup>,  $\varepsilon = 0.3796$  MeV and  $\Gamma = 8.5 \times 10^{-6}$  MeV, agree well with the old theoretical prediction.

In this work we investigate the amount of carbon and oxygen production in helium burning of red giants by slightly varying the nucleon-nucleon interaction. In Ref.<sup>7</sup> only hypothetical *ad hoc* shifts of the resonance energy of the  $0_2^+$ -state were investigated, whereas in this work we start by variations of the underlying N-N interaction.

After the Introduction we discuss in Sect. 2 our microscopic three-cluster model, the effective nucleon-nucleon (N-N) interactions, and the complex scaling method, which is used to describe the  $0_2^+$  state of  $^{12}\text{C}$ . In Sect. 3 the change of the triple-alpha reaction rates by slightly varying the underlying N-N interaction is calculated. In Sect. 4 we present the results for stellar model calculations for a low-mass, intermediate-mass and massive star using the triple-alpha reaction rates obtained with different strengths of the effective N-N interaction. In Sect. 5 the obtained results are summarized.

## 2 Nuclear physics

The astrophysical models that determine the amount of carbon and oxygen produced in red giant stars need some nuclear properties of  $^{12}\text{C}$  as input parameters. Namely, the position and width of the  $0_2^+$  resonance, which almost solely determines the triple-alpha reaction rate, and the radiative decay width for the  $0_2^+ \rightarrow 2_1^+$  transition in  $^{12}\text{C}$ . Here we calculate these quantities in a microscopic 12-body model.

In order to make such a calculation feasible, we use the microscopic cluster model. This approach assumes that the wave function of certain nuclei, like  $^{12}\text{C}$ , contain, with large weight, components which describe the given nucleus as a compound of 2-3 clusters. By assuming rather simple structures for the cluster internal states, the relative motions between the clusters, which are the most important degrees of freedom, can be treated rigorously. The strong binding of the free alpha-particle ( $^4\text{He}$ ) makes it natural that the low-lying states of  $^{12}\text{C}$  are largely  $3\alpha$ -structures<sup>8</sup>. Therefore, our cluster-model wave function for  $^{12}\text{C}$  looks like

$$\Psi^{12\text{C}} = \sum_{l_1, l_2} \mathcal{A} \left\{ \Phi^\alpha \Phi^\alpha \Phi^\alpha \chi_{[l_1 l_2] L}^{\alpha(\alpha\alpha)}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \right\}. \quad (1)$$

Here  $\mathcal{A}$  is the intercluster antisymmetrizer, the  $\Phi^\alpha$  cluster internal states are translationally invariant  $0s$  harmonic-oscillator shell-model states with zero total spin, the  $\boldsymbol{\rho}$  vectors are the intercluster Jacobi coordinates,  $l_1$  and  $l_2$  are the angular momenta of the two relative motions,  $L$  is the total orbital angular momentum and  $[\dots]$  denotes angular momentum coupling. The total spin and parity of  $^{12}\text{C}$  are  $J = L$  and  $\pi = (-1)^{l_1+l_2}$ , respectively.

We want to calculate the resonance energy of the  $0_2^+$  state in  $^{12}\text{C}$ , relative to the  $3\alpha$ -threshold, and the  $0_2^+ \rightarrow 2_1^+$  radiative (E2) width, while slightly varying the strength of the effective nucleon-nucleon (N-N) interaction. This way, the sensitivity of the carbon and oxygen production on the effective N-N interaction can be studied. The  $0_2^+$  state is situated above the  $3\alpha$ -threshold, therefore for a rigorous description one has to use an approach which can describe three-body resonances correctly. We choose the complex scaling method<sup>9</sup> that has already been used in a variety of other nuclear physics problems, see e.g.<sup>10</sup>. In this method the eigenvalue problem of a transformed Hamiltonian

$$\hat{H}_\theta = \hat{U}(\theta) \hat{H} \hat{U}^{-1}(\theta) \quad (2)$$

is solved, instead of the original  $\hat{H}$ , where the transformation  $\hat{U}$  acts on a function  $f(\mathbf{r})$  as

$$\hat{U}(\theta) f(\mathbf{r}) = e^{3i\theta/2} f(\mathbf{r}e^{i\theta}). \quad (3)$$

Resonances are eigenvalues of  $\widehat{H}$  with  $E_{\text{res}} = \varepsilon - i\Gamma/2$  ( $\varepsilon, \Gamma > 0$ ) complex energy, where  $\varepsilon$  is the position of the resonance while  $\Gamma$  is the full width. A straightforward description of such states are difficult because their wave functions are exponentially divergent in the asymptotic region. The effect of the Eq. (2) complex scaling transformation is that the positive-energy continuum of  $\widehat{H}$  gets rotated (by the angle  $2\theta$ ) down into the complex energy plane, while the wave function of any resonance becomes square-integrable if  $2\theta > |\arg E_{\text{res}}|$ . Using this method<sup>8</sup>, we were able to localize the energy of the  $0_2^+$  state of  $^{12}\text{C}$ .

The other important quantity that needs to be calculated is the radiative width of the  $0_2^+$  state, coming from the electric dipole (E2) decay into the  $2_1^+$  state of  $^{12}\text{C}$ . This calculation involves the evaluation of the E2 operator between the initial  $0_2^+$  three-body scattering state and the final  $2_1^+$  bound state<sup>11</sup>. The proper three-body scattering-state treatment of the  $0_2^+$  initial state is not feasible for the time being, therefore we use a bound-state approximation to it. This is an excellent approximation for the calculation of  $\Gamma_\gamma$  because the total width of the  $0_2^+$  state is very small (8.5 eV<sup>6</sup>). The value of  $\Gamma_\gamma$  is rather sensitive to the energy difference between the  $0_2^+$  and  $2_1^+$  states, so we have to make sure that the experimental energy difference is correctly reproduced.

In order to see the dependence of the results on the chosen effective N-N interaction, we performed the calculations using three different forces. The Minnesota (MN)<sup>12</sup>, and the rather different Volkov 1 and 2 (V1, V2)<sup>13</sup> forces achieve similar quality in describing light nuclear systems. We slightly adjusted a parameter (the exchange mixture) of each forces in order to get the  $0_2^+$  resonance energy right at the experimental value. This leads to  $u = 0.941$  for MN and  $m = 0.568$  and  $m = 0.594$  for V1 and V2, respectively. The results coming from these forces are our baseline predictions. Then, we multiplied the strengths of the forces by a factor  $p$ , which took up the values  $p = 0.996, 0.998, 0.999, 1.001, 1.002, \text{ and } 1.004$ , respectively, and calculated the resonance energies and gamma widths again. This way we can monitor the sensitivity of the  $^{12}\text{C}$  properties, and thus the carbon and oxygen production in stellar environments, as the function of the N-N interaction strength. The results for the resonance energies and widths are shown in Table 1.

As we mentioned, we correctly reproduced the  $0_2^+ - 2_1^+$  energy difference for  $p = 1.0$ . This required the use of a slightly different force for  $2_1^+$ . The resulting  $\Gamma_\gamma$  values should be compared to the experimental figure,  $3.7 \pm 0.5 \text{ meV}^6$ . Our model performs well, considering the fact that no effective charges were used in the calculations. Using an effective charge  $e_{\text{eff}}$  for both the protons and neutrons would lead to  $\Gamma_\gamma$  multiplied by  $(1 + 2e_{\text{eff}})^2$ .

Table 1: The energy,  $E_r$  (in keV), and gamma width,  $\Gamma_\gamma$  (in meV), of the  $0_2^+$ -resonance as a function of the factor  $p$

N-N interaction	MN		V1		V2	
$p$	$\varepsilon(p)$	$\Gamma_\gamma(p)$	$\varepsilon(p)$	$\Gamma_\gamma(p)$	$\varepsilon(p)$	$\Gamma_\gamma(p)$
1.004	273.3	2.85	294.4	3.21	306.0	3.28
1.002	327.5	2.91	337.5	3.26	343.7	3.31
1.001	353.7	2.93	358.7	3.27	361.7	3.32
1.000	379.6	2.96	379.6	3.29	379.6	3.33
0.999	405.2	2.99	400.3	3.31	397.2	3.34
0.998	430.5	3.01	420.8	3.32	414.6	3.34
0.996	481.4	3.07	460.7	3.34	450.0	3.34

### 3 Triple-alpha reaction rate

The results shown in Table 1 can be used to calculate the triple-alpha reaction rates. The reaction rate for the triple-alpha process proceeding via the ground state of  $^8\text{Be}$  and the  $0_2^+$ -resonance in  $^{12}\text{C}$  is given by<sup>14</sup>

$$r_{3\alpha} = 3^{\frac{3}{2}} N_\alpha^3 \left( \frac{2\pi\hbar^2}{M_\alpha k_B T} \right)^3 \frac{\omega\gamma}{\hbar} \exp\left(-\frac{\varepsilon}{k_B T}\right), \quad (4)$$

where  $M_\alpha$  and  $N_\alpha$  is the mass and the number density of the  $\alpha$ -particle, respectively. The temperature of the stellar plasma is given by  $T$ . The quantity  $\varepsilon$  denotes the difference in energy between the  $0_2^+$ -resonance in  $^{12}\text{C}$  and the  $3\alpha$ -particle threshold. The resonance strength  $\omega\gamma$  is given by

$$\omega\gamma = \frac{\Gamma_\alpha \Gamma_{\text{rad}}}{\Gamma_\alpha + \Gamma_{\text{rad}}} \approx \Gamma_\gamma. \quad (5)$$

The approximation of the above expression for the decay widths of the  $0_2^+$ -resonance follows, because for the  $\alpha$ -width  $\Gamma_\alpha$ , the radiation width  $\Gamma_{\text{rad}}$ , the electromagnetic decay width  $\Gamma_\gamma$  to the first excited state of  $^{12}\text{C}$ , and the electron-positron pair emission decay width  $\Gamma_{\text{pair}}$  into the ground state of  $^{12}\text{C}$  the following approximations hold: (i)  $\Gamma_\alpha \gg \Gamma_{\text{rad}}$  and (ii)  $\Gamma_{\text{rad}} = \Gamma_\gamma + \Gamma_{\text{pair}} \approx \Gamma_\gamma$ . Therefore, Eq. (4) can be approximated by:

$$r_{3\alpha} \approx 3^{\frac{3}{2}} N_\alpha^3 \left( \frac{2\pi\hbar^2}{M_\alpha k_B T} \right)^3 \frac{\Gamma_\gamma}{\hbar} \exp\left(-\frac{\varepsilon}{k_B T}\right), \quad (6)$$

The two quantities in Eq. (6) that change their value by varying the effective N-N interaction are the energy of the  $0_2^+$ -resonance  $\varepsilon$  in  $^{12}\text{C}$  and its

electromagnetic decay width  $\Gamma_\gamma$ . However, the change in the reaction rate by varying the effective N-N interaction with  $\varepsilon$  in the exponential factor of Eq. (6) is much larger than the linear change with  $\Gamma_\gamma$ .

#### 4 Stellar burning

The significance of low-mass (with masses less than about  $2 M_\odot$ , intermediate-mass (between about  $2M_\odot$  and  $10M_\odot$ ), and massive stars (with masses more than about  $10M_\odot$ ) in the nucleosynthesis of carbon is still not quite clear<sup>15</sup>. Some authors claim that low-mass and intermediate-mass stars dominate in the production of carbon, whereas others favor the production of carbon in massive stars. In a recent investigation<sup>15</sup> by spectral analysis of solar-type stars in the Galactic Disk, the results are consistent with carbon production in massive stars but inconsistent with a main origin of carbon in low-mass stars. The significance of intermediate-mass stars for the production of carbon in the Galaxy is still somewhat unclear. Therefore we performed a stellar model calculation for a typical low-mass, intermediate-mass, and massive star, respectively.

The calculation of the stellar models are performed with a contemporary stellar evolution code, which contains the latest physics input. In particular, using this code, up-to-date solar models<sup>16</sup> can be produced as well as the evolution of low-mass stars can be followed through the thermal pulses of asymptotic giant branch-(AGB) stars<sup>17</sup>. The nuclear network is designed preferentially to calculate the hydrogen and helium burning phases in low-mass stars. Additionally, the basic reactions of carbon and oxygen burning are included, which may destroy the produced carbon and oxygen in massive stars.

In this work the evolution of a 1.3, 5, and 20 solar mass star is calculated, which should represent the typical evolution of low-mass, intermediate-mass, and massive stars, respectively. The stars are followed from the onset of hydrogen burning until the first thermal pulses in the AGB, or until the core temperature reaches  $10^9$  K in the case of the  $20M_\odot$  star, as the nuclear network is not sufficient to go beyond this phase.

Large portions of the initial mass of a star are returned to the interstellar medium (ISM) through stellar winds basically during the thermal pulse phase. Unfortunately, basically due to the simple convection model used in stellar modeling, the composition of the wind can not yet be determined very accurately from stellar evolution theory. However, it is beyond the scope of the present investigations to determine how and when the material is return back to the ISM. Instead we investigate how much C and O is produced altogether

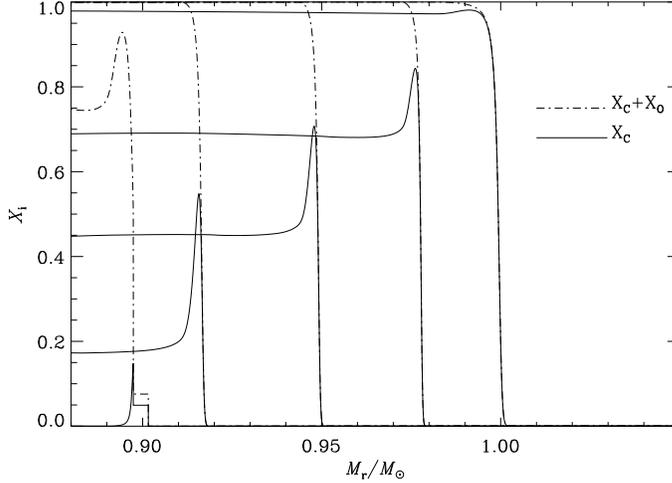


Figure 1: The radial abundance profile of carbon and oxygen within a  $5.0 M_{\odot}$  shortly before the first thermal pulse. The quantities  $X_C$  and  $X_O$  denote the radial abundances of carbon and oxygen, respectively. The models are from left to right with  $p = 0.996, 0.999, 1.00, 1.001$  and  $1.004$ .

in the star, which then may be blown away by stellar winds.

For the three masses quoted above, the evolution of stars is calculated using the predictions of the MN interaction with  $p = 0.996, 0.999, 1.000, 1.001,$  and  $1.004$ . In Fig. 1 the radial abundance profiles of C and O are shown for a  $5M_{\odot}$  star, shortly before the first thermal pulse. With the standard cross section for the triple-alpha reaction (He-burning front at  $M_r = 0.95M_{\odot}$ ) roughly the same amount of carbon and oxygen is produced. Changing the N-N interaction strength by only about 0.1%, this ratio could already be altered significantly. If the N-N interaction strength is enhanced or reduced by 0.4%, respectively, then almost no oxygen or carbon is produced in a  $5M_{\odot}$  star. Similar behavior can be observed for the  $1.3M_{\odot}$  star, while in the case of a  $20M_{\odot}$  star, the carbon can be destroyed almost totally even for  $p = 0.999$ .

Thus, even with a minimal change of 0.4% in the strength of the N-N force, carbon-based life appears to be impossible, since all the stars then would produce either almost solely carbon or oxygen, but could not produce both elements. For smaller variations in the N-N interaction the development of life depends on the stellar population, as for instance a  $20M_{\odot}$  star with  $p = 0.999$

does not produce carbon, while the  $1.3M_{\odot}$  and  $5M_{\odot}$  stars still do.

## 5 Summary

We have investigated the change of the carbon and oxygen production in the helium burning of low-mass, intermediate-mass and massive stars by varying the underlying nucleon-nucleon interaction. A slight variation of the nucleon-nucleon interaction leads to a change of the resonance energy of the second  $0^+$  state ( $0_2^+$ ) in  $^{12}\text{C}$ , thus drastically modifying the of the triple-alpha reaction rate. The changes in the relevant nuclear parameters in the triple-alpha reaction rate, e.g. the resonance energy and radiative decay width of the  $0_2^+$  state in  $^{12}\text{C}$ , were obtained with the help of a microscopic three-cluster model.

The impact of changing the triple-alpha reaction rate by varying the effective N-N interaction in the evolution of 1.3, 5 and 20 solar mass stars is calculated by following the typical evolution of a low-mass, intermediate-mass, and massive star, respectively. The calculations were carried out with a contemporary stellar evolution code, which contains the latest physics input. The result is that even with a small change of 0.4% in the N-N interaction strength, carbon-based life appears to be impossible, since all the stars then would produce either almost solely carbon or oxygen, but could not produce both elements.

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