S-matrix studies of resonances in A=3, 4, 5, 6, and 12 nucleon systems *

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Model

Microscopic **Resonating Group Model** (RGM) with two/three-cluster dynamics for

$$^3n = n + n + n, \ \ ^3p = p + p + p$$
$$^4\mathrm{He} = \{t + p, h + n\}$$
$$^5\mathrm{He} = \alpha + n, \ \ ^5\mathrm{Li} = \alpha + p,$$
$$^6\mathrm{He} = \alpha + n + n, \ \ ^6\mathrm{Li} = \alpha + p + n, \ \ ^6\mathrm{Be} = \alpha + p + p$$
$$^{12}\mathrm{C} = \alpha + \alpha + \alpha$$

Here $\alpha = {}^4\mathrm{He}$, $t = {}^3\mathrm{H}$ and $h = {}^3\mathrm{He}$.

Wave functions:

$$\Psi_2 \ = \ \sum_{L,S} \mathcal{A}\Big\{\Big[[\Phi^A\Phi^B]_S \chi_L(\boldsymbol{\rho})\Big]_{JM}\Big\},$$

$$\Psi_3 \ = \ \sum_{l_1,l_2,L,S} \mathcal{A}\Big\{\Big[[\Phi^A\Phi^B\Phi^C]_S \chi_{[l_1,l_2]L}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\Big]_{JM}\Big\},$$

$\mathcal{A}$ is the intercluster antisymmetrizer, Φ are translationally invariant $0s$ harmonic-oscillator shell-model states, $\boldsymbol{\rho}$ are intercluster relative coordinates, l_1, l_2, L are orbital angular momenta, S is the total spin. In Ψ_3 all possible sets of relative coordinates $[A(BC), C(AB), B(AC)]$ and angular momentum couplings are included.

χ are the unknown relative motions (the most important degrees of freedom).

Localization of S-matrix poles at complex energies

For two-cluster systems: the Schrödinger equation is solved for the relative motion at complex energies with $\rho \to \infty$

$$\chi(\varepsilon, \rho) \to H^-(k\rho) - \tilde{S}(\varepsilon) H^+(k\rho) \quad \text{for} \quad \rho \to \infty.$$

ε and k are the *complex* energies and wave numbers of the relative motions, and H^- and H^+ are the incoming and outgoing Coulomb functions.

For three-cluster systems: the eigenvalue problem of a new Hamiltonian

$$\widehat{H}_\theta \;=\; \widehat{U}(\theta)\widehat{H}\widehat{U}^{-1}(\theta),$$

is solved, where $\widehat{H}$ is the original many-body Hamiltonian and $\widehat{U}$ is the complex scaling transformation acting on each dynamical coordinate (relative motion): $\widehat{U}(\theta)f(\mathbf{r}) = e^{3i\theta/2}f(\mathbf{r}e^{i\theta})$.
The spectrum of $\widehat{H}_\theta$: the continuum is rotated by 2θ relative to the real energy axis, plus possibly a few isolated complex points at the resonance and bound state poles (independent of θ).

Model independent approach for two-cluster systems: R matrix fit of experimental data $\Longrightarrow$ construction of S from $R \Longrightarrow$ search for the poles of S

$$\boxed{\varepsilon = E_r - i\Gamma/2}$$

Results

A=3 (Phys. Rev. C**53**, 1589 (1996)):

Motivation: Pion double charge exchange (DCX) experiments on ^{3}He found indications of a possible $3n$ resonance.

Result: All $3n$ partial waves up to $J = 5/2$ are nonresonant except $J^\pi = 3/2^+$. For $3/2^+$:

$$3n: \quad E_r = 14 \text{ MeV}, \;\; \Gamma = 13 \text{ MeV}$$
$$3p: \quad E_r = 15 \text{ MeV}, \;\; \Gamma = 14 \text{ MeV}$$

A=4 (Phys. Rev. C**55**, 2366 (1997)):

Motivation: The nature of the first excited state of ^{4}He (0_2^+) is not well understood in shell-models and conventional R-matrix models.

Result: Parameters of the 0_2^+ state of ^{4}He from the S-matrix pole positions:

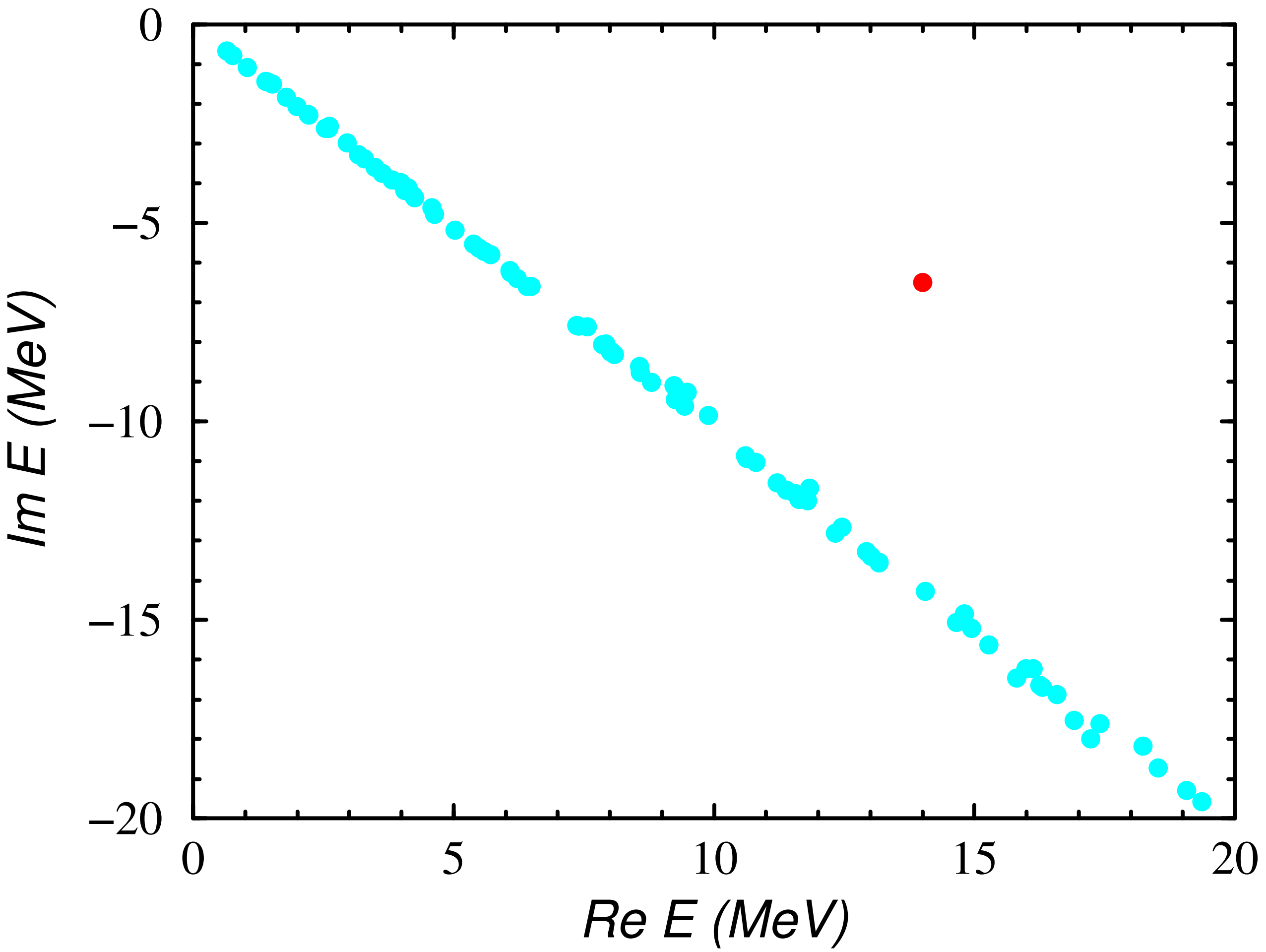
$$\text{RGM}: \;\; E_r = 93 \text{ keV}, \;\; \Gamma = 390 \text{ keV}$$
$$R\text{−matrix}: \;\; E_r = 114 \text{ keV}, \;\; \Gamma = 392 \text{ keV}$$

A=5 (Phys. Rev. C**55**, 536 (1997)):

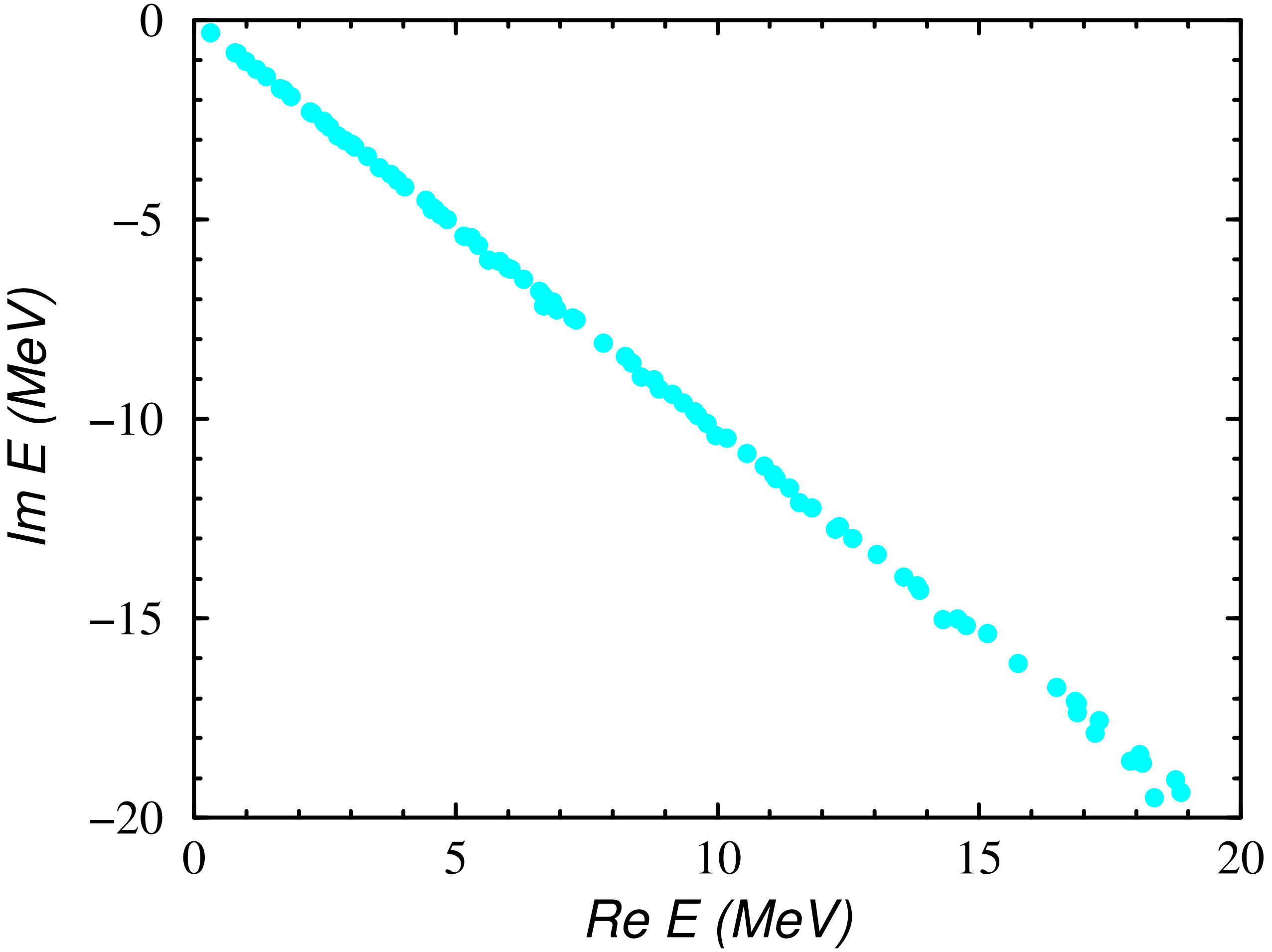
Motivation: The empirical parameters of the $3/2^-$ and $1/2^-$ resonances of ^{5}He and ^{5}Li are in contradiction with those that appear, e.g., in neutron halo studies of ^{6}He (especially for the SO splitting).

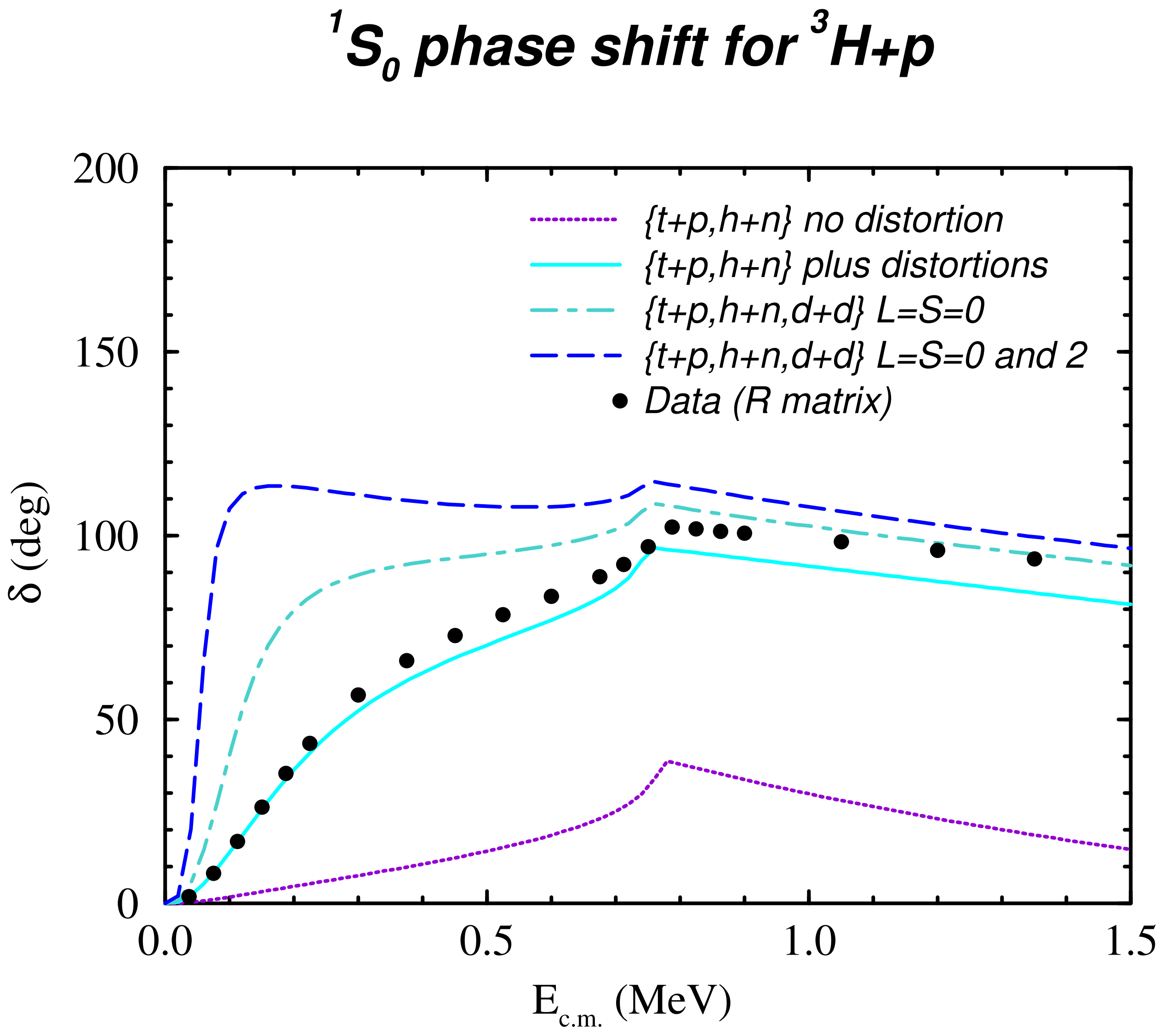
Result: RGM and extended R matrix agree on resonance parameters. The $3/2^-$ and $1/2^-$ pole trajectories rule out a > 2 MeV SO splitting for ^{5}He.

Eigenvalues of H_θ for $J^\pi=3/2^+$ ^3n states



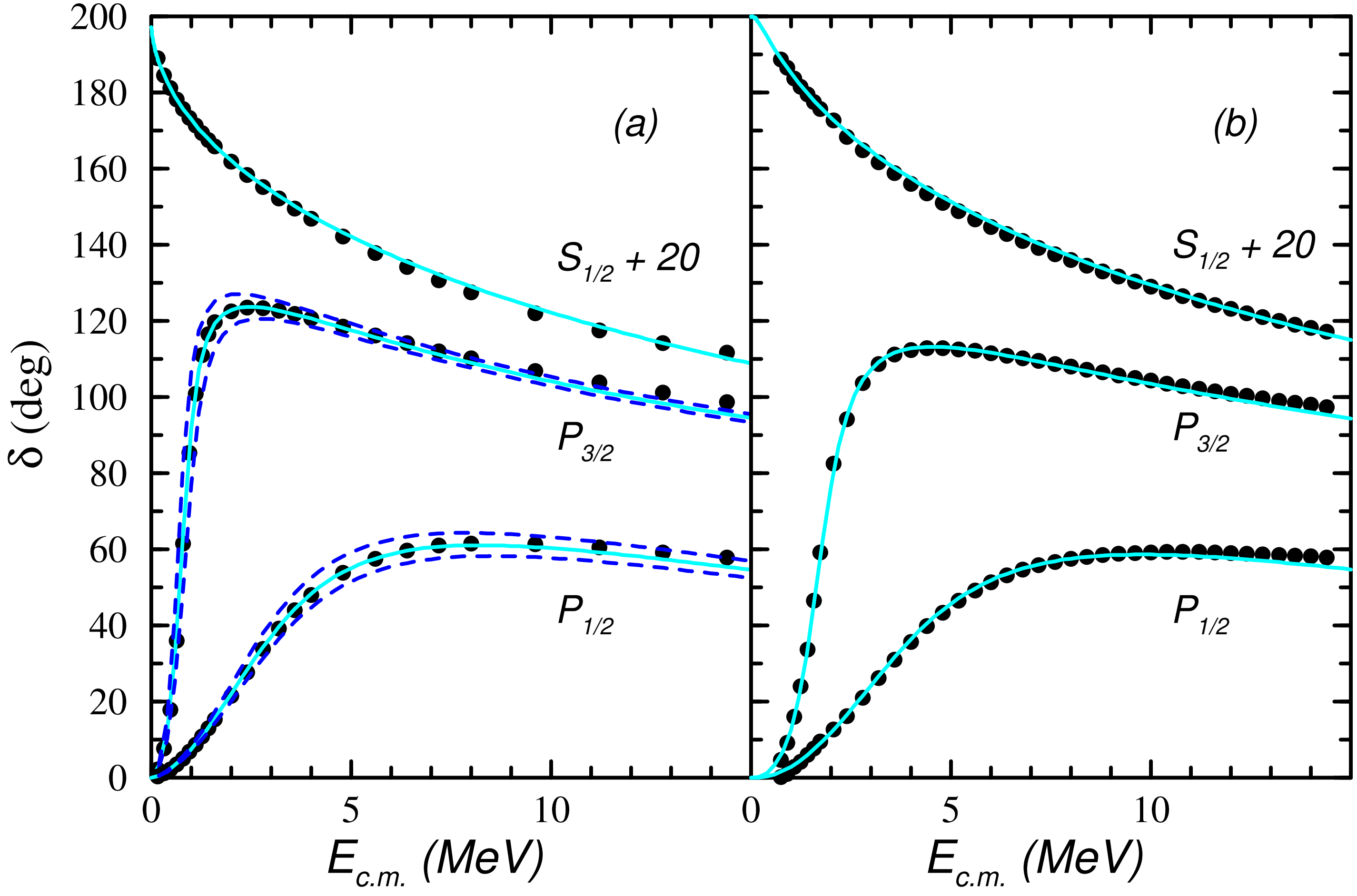
Eigenvalues of H_θ for $J^\pi = 3/2^-$ ^3n states





$^1S_0$ phase shift for ^3H+p
{t+p,h+n} no distortion
{t+p,h+n} plus distortions
{t+p,h+n,d+d} L=S=0
{t+p,h+n,d+d} L=S=0 and 2
Data (R matrix)
δ (deg)
$E_{c.m.}$ (MeV)
0
50
100
150
200
0.0
0.5
1.0
1.5

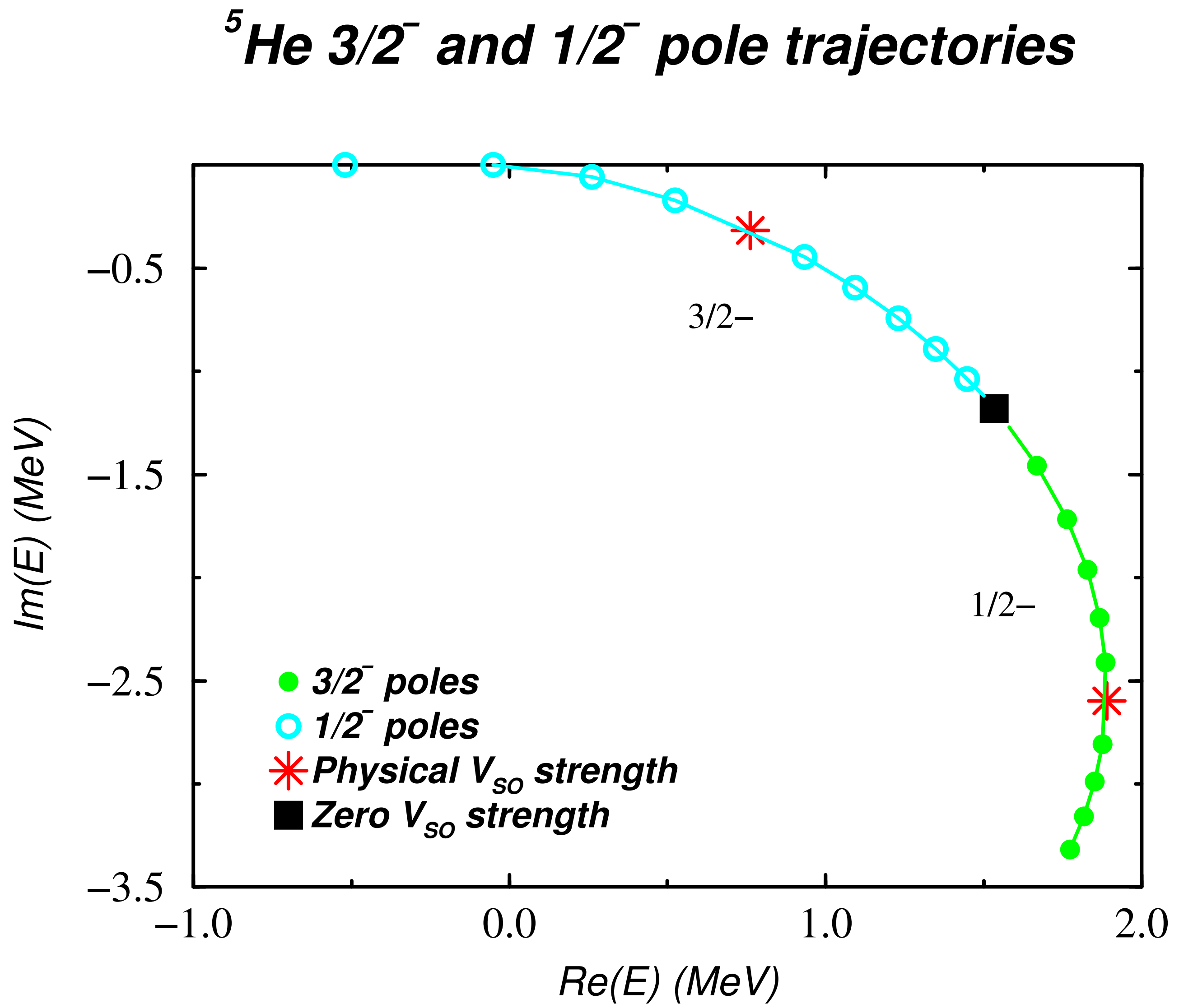
α+n and α+p phase shifts



TABLES

TABLE I. Parameters of the low-energy ^{5}He and ^{5}Li resonances in the center-of-mass frame. E_r is the resonance position relative to the $\alpha + N$ threshold, and Γ is the full width at half maximum. All numbers are in MeV.

	^{5}He				^{5}Li			
Method	$E_r(3/2^-)$	$\Gamma(3/2^-)$	$E_r(1/2^-)$	$\Gamma(1/2^-)$	$E_r(3/2^-)$	$\Gamma(3/2^-)$	$E_r(1/2^-)$	$\Gamma(1/2^-)$
Compilation [14]	0.89±0.05	0.60±0.02	4.89±1	4±1	1.96±0.05	≈ 1.5	7–12	5±2
R-matrix, stripping [13]	0.838±0.018	0.645±0.046	2.778±0.46	3.6±1.2	1.76±0.06	1.18±0.13	3.63±0.56	4.1±2.5
R-matrix, pickup [13]	0.869±0.003	0.723±0.019	3.449±0.4	5.3±2.3	1.86±0.01	1.44±0.08	4.54±0.5	6.1±2.8
Scattering ampl. [5]	0.778	0.639	1.999	4.534	1.637	1.292	2.858	6.082
S-matrix, RGM	0.76	0.63	1.89	5.20	1.67	1.33	2.70	6.25
Extended R-matrix	0.80	0.65	2.07	5.57	1.69	1.23	3.18	6.60



⁵He 3/2⁻ and 1/2⁻ pole trajectories
Im(E) (MeV)
Re(E) (MeV)
−0.5
−1.5
−2.5
−3.5
−1.0
0.0
1.0
2.0
3/2−
1/2−
3/2⁻ poles
1/2⁻ poles
Physical V_so strength
Zero V_so strength

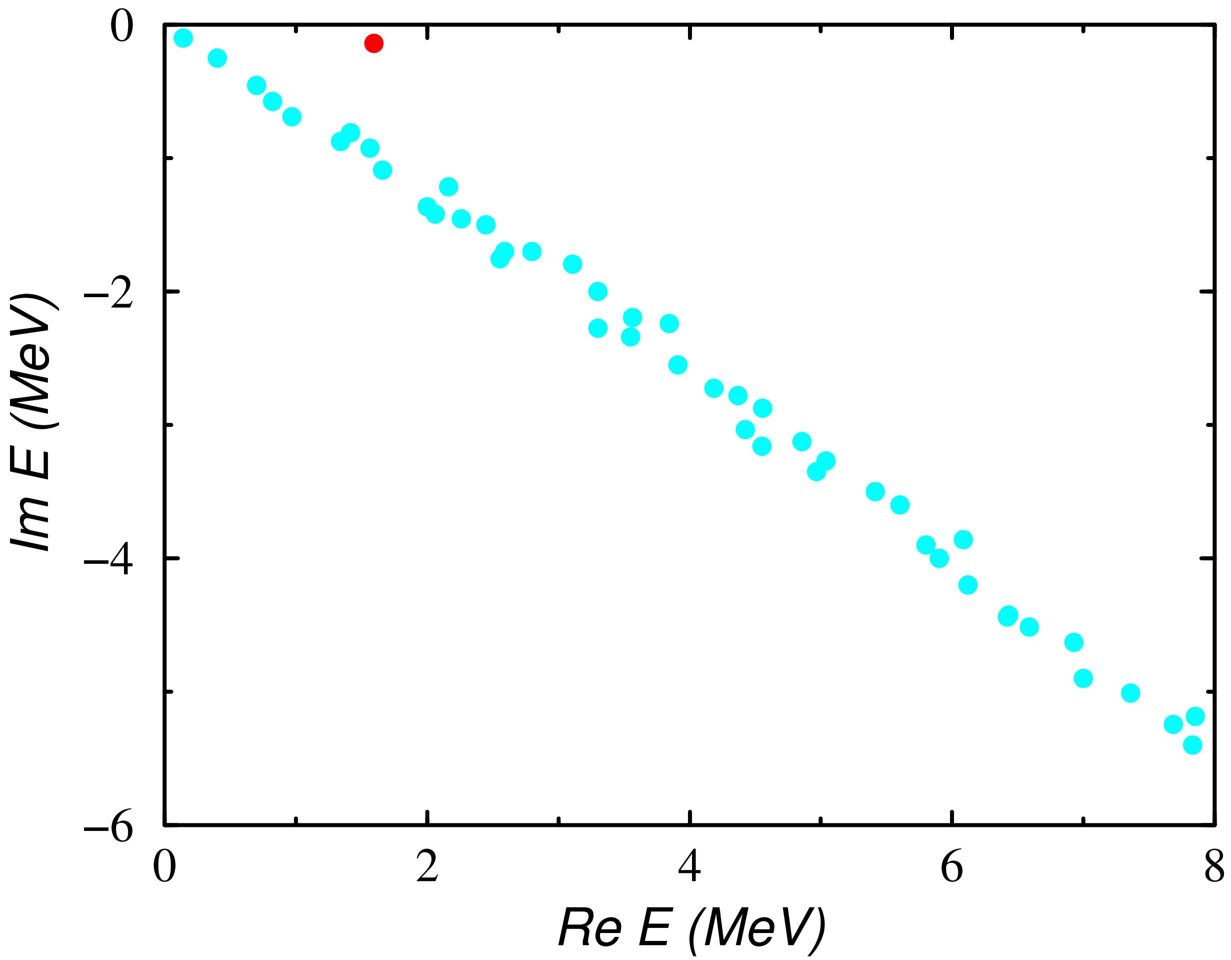
A=6 (Phys. Rev. C**49**, 3035 (1994)):

Motivation: A new type of collective excitation, the **soft dipole mode** was suggested for neutron halo nuclei, like ^{6}He. Searching for 1^- resonance poles in ^{6}He.

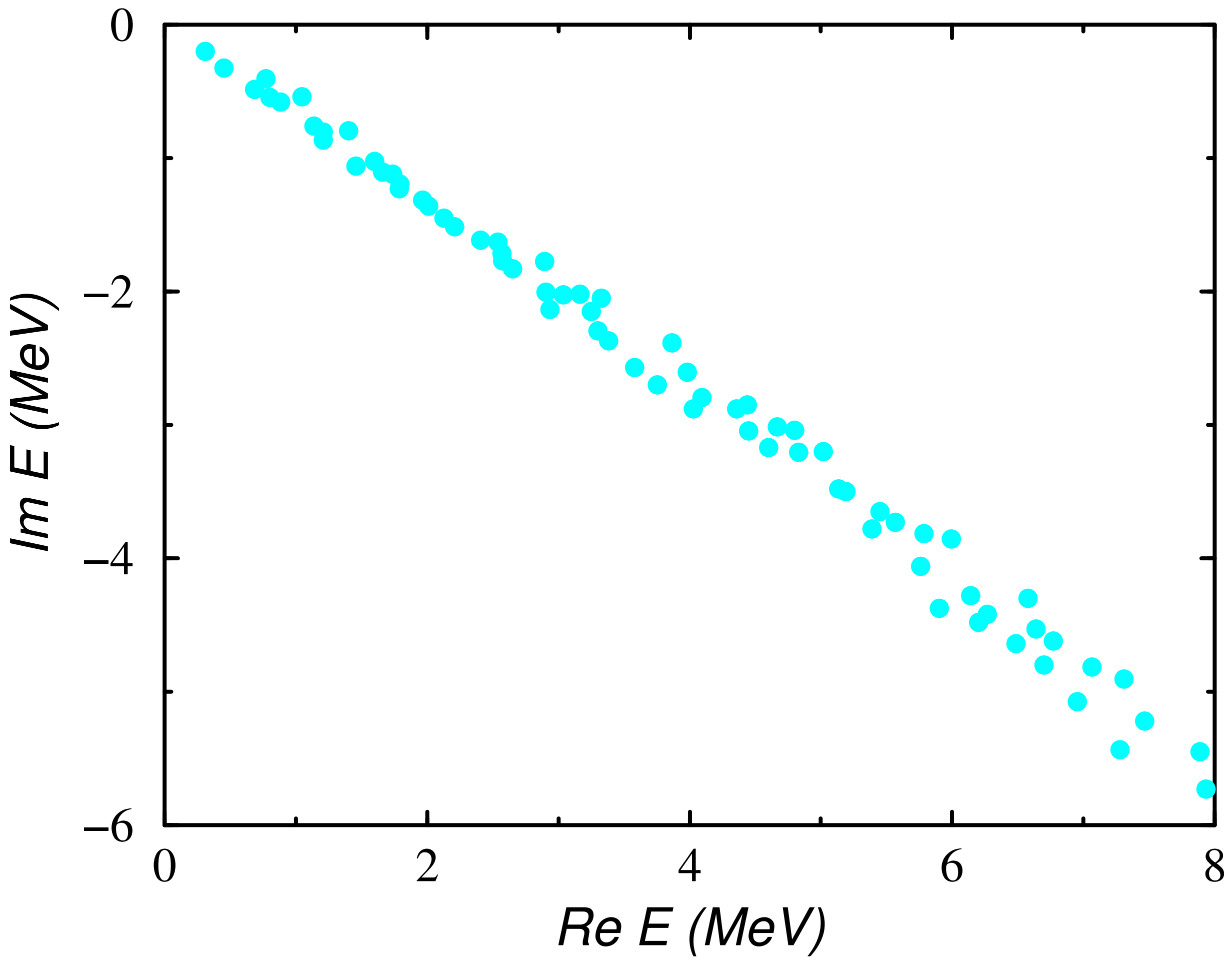
Result: All experimentally known low-lying three-body resonances are found in ^{6}He, ^{6}Li, and ^{6}Be. No low-lying 1^- state was found. The soft dipole mode, if exists, is not a three-body resonance in ^{6}He.

		Theory		Experiment	
	J^π, T	E (MeV)	Γ (MeV)	E (MeV)	Γ (MeV)
^{6}He	$2^+, 1$	0.74	0.06	0.822±0.025	0.113±0.020
^{6}Li	$0^+, 1$	0.22	0.001	−0.137	0
	$2^+, 0$	not extracted		0.610±0.022	1.7±0.2
	$2^+, 1$	1.59	0.28	1.696±0.015	0.54±0.020
	$1^+, 0$	5.71	3.89	1.95±0.05	1.5±0.2
^{6}Be	$0^+, 1$	1.52	0.16	1.371	0.092±0.006
	$2^+, 1$	2.81	0.87	3.04±0.05	1.16±0.06

Eigenvalues of H_θ for $J^\pi,T=2^+,1$ ^{6}Li states



Eigenvalues of H_θ for $J^\pi=1^-$ ^6He states

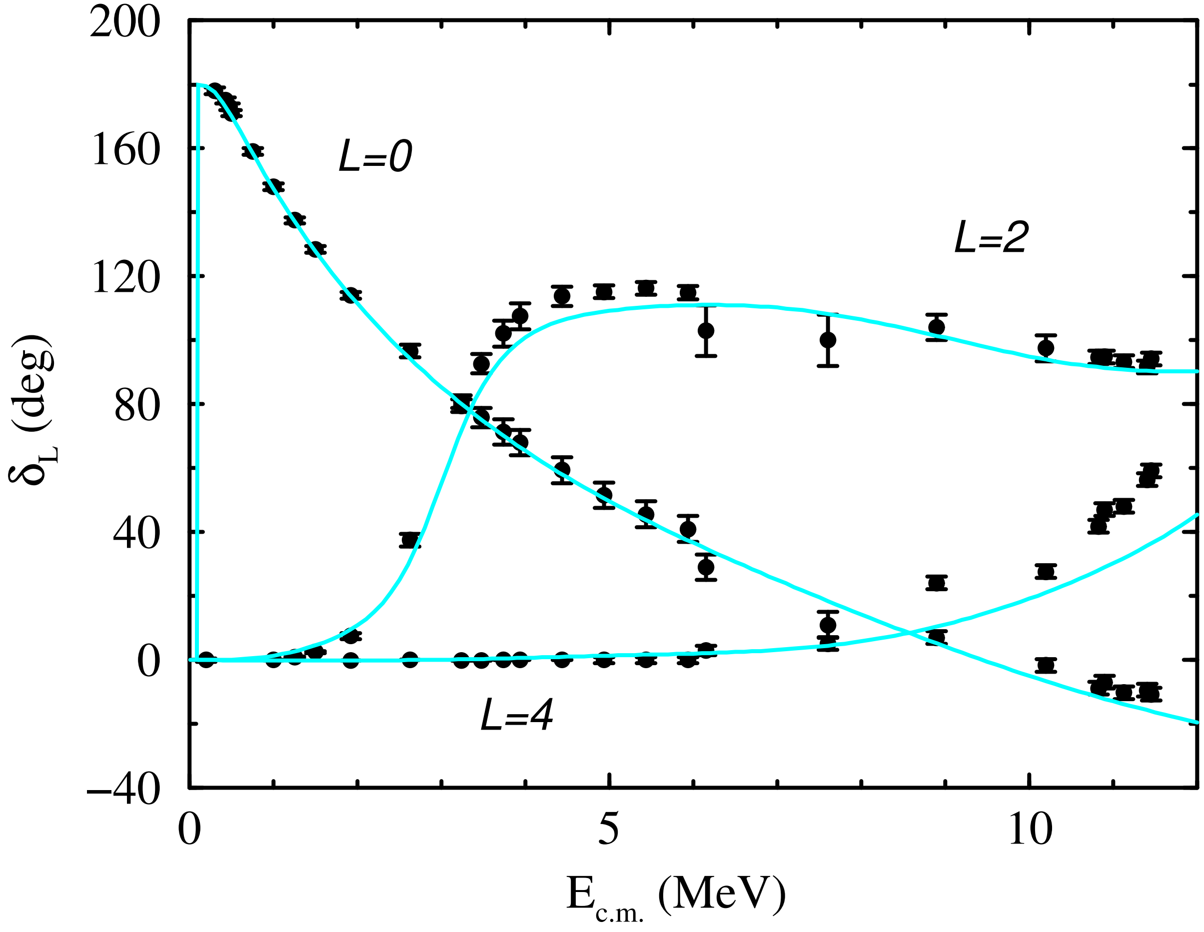


A=12 (Nucl. Phys. A**618**, 55 (1997)):

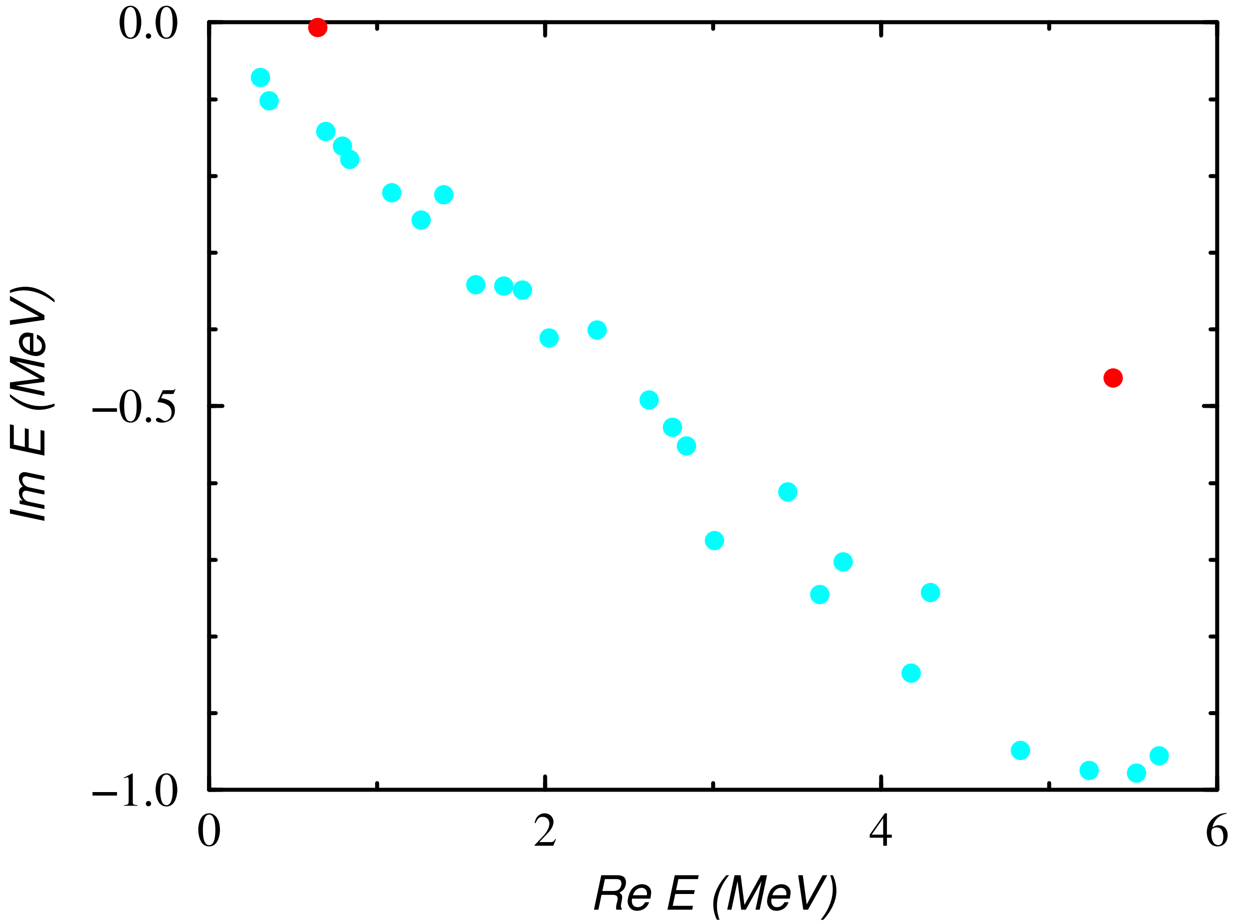
Motivation: The 0_2^+ state plays a very important role in stellar nucleosynthesis. It was assumed to be an intrinsic state of ^{12}C. Is this state a three-alpha resonance?

Result: All experimentally known low-lying natural-parity three-alpha resonances are found in ^{12}C. The 0_2^+ state is really a three-alpha resonance.

α+α *phase shifts*



Eigenvalues of H_θ for $J^\pi=0^+$ ^{12}C states



Summary

- The exploration of resonances through the analytic structure of the S matrix is proven to be very fruitful
- Good general agreement between the results of the RGM and the extended R-matrix methods
- We recommend using the complex S-pole prescription to specify resonance parameters in all cases
- Application to other systems is in progress