S-matrix studies of resonances in $A = 3$, 4, 5, 6, and 12 nucleon systems

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Resonances of certain light nuclei are explored by studying the complex pole structures of the scattering matrices. Among other results we predict the existence of three-neutron and three-proton resonances, a small spin-orbit splitting in the low-lying ^{5}He and ^{5}Li states, and the nonexistence of the soft dipole resonance in ^{6}He.

1. Introduction

The investigation of resonance structures in light nuclei offers a rich source of information on many-body dynamics, nucleon-nucleon interaction, shell-structure, etc. Conventionally, resonances are studied both experimentally and theoretically by analyzing certain observables (cross sections, scattering amplitudes, phase shifts, etc.) at *real energies* using the methods and concepts of scattering theory. However, in scattering theory resonances are defined through the analytic properties of certain quantities (S matrix, Fredholm determinant, Jost function) at *complex energies.* We believe that the theoretical study of few-nucleon scattering at complex energies offers a unique insight into the dynamics of these problems.

We study selected resonances of $A = 3$, 4, 5, 6, and 12 nucleon systems by exploring the analytic properties of the S matrices at complex energies. The nuclei are described within the Resonating Group Model (RGM), assuming 2- or 3-cluster dynamics. Certain S-matrix poles are also extracted from R-matrix fits of experimental data by extending the S matrix, generated from the R matrix, to complex energies.

2. Localization of S-matrix poles at complex energies

We use a microscopic two/three-cluster description of the $^3n(=n{+}n{+}n)$, $^3p(=p{+}p{+}p)$, ^{4}He$(=\{t+p,h+n\})$, ^{5}He$(=\alpha+n)$, ^{5}Li$(=\alpha+p)$, ^{6}He$(=\alpha+n+n)$, ^{6}Li$(=\alpha+p+n)$, ^{6}Be$(=\alpha+p+p)$, and ^{12}C$(=\alpha+\alpha+\alpha)$ nuclei. Here $\alpha={}^4$He, $t={}^3$H, $h={}^3$He, and the cluster structures, assumed in the model, are indicated. We assume simple $0s$ harmonic oscillator shell-model wave functions for the internal states of the clusters (α, t, and h). However, the relative motions between the clusters, which are the most important degrees of freedom, are treated with the rigor of few-body physics.

The wave functions of the two- and three-cluster systems look like

$$\Psi = \sum_{L,S} \mathcal{A}\left\{ \left[\left[\Phi^A \Phi^B \right]_S \chi_L(\boldsymbol{\rho}) \right]_{JM} \right\}, \tag{1}$$

and

$$\Psi = \sum_{l_1,l_2,L,S} \mathcal{A}\left\{\left[\left[\Phi^A\Phi^B\Phi^C\right]_S \chi_{[l_1,l_2]L}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\right]_{JM}\right\}, \tag{2}$$

respectively. Here $\mathcal{A}$ is the intercluster antisymmetrizer, the Φ cluster internal states are translationally invariant $0s$ harmonic-oscillator shell-model states, the $\boldsymbol{\rho}$ vectors are the intercluster relative coordinates, l_1 and l_2 are the angular momenta of the two relative motions, L is the total orbital angular momentum, S is the total spin, and $[\ldots]$ denotes angular momentum coupling. In the case of three-cluster dynamics all possible sets of relative coordinates $[A(BC),\ C(AB),\ B(AC)]$ and angular momentum couplings are included in (2).

Putting (1) or (2) into the N-body Schrödinger equation we get equations for the unknown relative motion functions χ. For two-body (three-body) bound states they are expanded in terms of (products of) Gaussian functions, and the expansion coefficients are determined from a variational principle for the energy. For two-body scattering states the χ functions are expanded in terms of Gaussian functions matched with the correct asymptotics, and the expansion coefficients are determined from the Kohn-Hulthén variational method for the S matrix [1].

In scattering theory resonances are defined as complex-energy solutions of the Schrödinger equation that correspond to the poles of the S matrix (or equivalently the zeros of the Fredholm determinant or Jost function). In order to obtain these complex solutions, we implemented a direct analytic continuation of the S matrix for two-cluster systems [2,3] and the complex scaling method for three-cluster systems [4–6].

For two-cluster systems we solve the Schrödinger equation for the relative motion at complex energies with the the following boundary condition for $\rho \to \infty$

$$\chi(\varepsilon,\rho) \to H^-(k\rho) - \tilde{S}(\varepsilon)H^+(k\rho). \tag{3}$$

Here ε and k are the *complex* energies and wave numbers of the relative motions, and H^- and H^+ are the incoming and outgoing Coulomb functions, respectively. The function $\tilde{S}$ has no physical meaning, except if it is singular at the energy ε. Then $\tilde{S}$ coincides with the physical S matrix describing a purely outgoing solution, that is a resonance. So we search for the poles of $\tilde{S}$ at complex energies and extract the resonance parameters from $\varepsilon = E_{\mathrm{r}} - i\Gamma/2$ [7].

For the three-cluster systems we solve the eigenvalue problem of a new Hamiltonian defined by

$$\widehat{H}_\theta = \widehat{U}(\theta)\widehat{H}\widehat{U}^{-1}(\theta), \tag{4}$$

where $\widehat{H}$ is the original many-body Hamiltonian and $\widehat{U}$ is the complex scaling transformation which acts on a function $f(\mathbf{r})$ as $\widehat{U}(\theta)f(\mathbf{r}) = e^{3i\theta/2}f(\mathbf{r}e^{i\theta})$. In the case of a multicluster system the transformation is performed on each dynamical coordinate (relative motion). The solution of the complex-scaled Schrödinger equation results in a spectrum with continuum cuts rotated by 2θ relative to the real energy axis plus possibly a few isolated complex points at the resonance and bound state poles [8].

We can perform a similar analysis of the analytic properties of the S matrices in a model-independent way starting from the experimental data. The data can be described

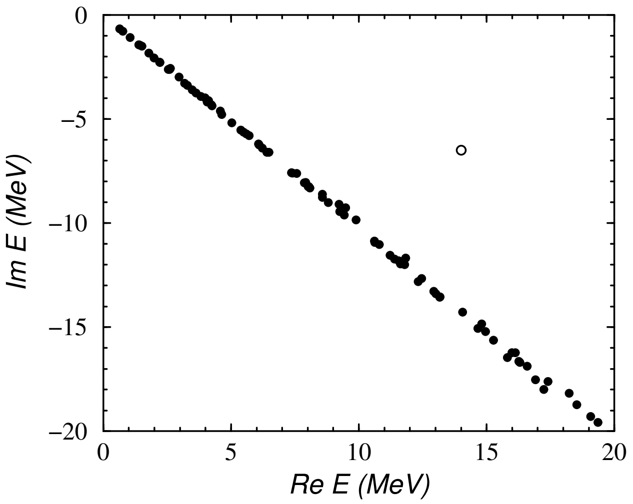

Figure 1. Energy eigenvalues of the complex scaled Hamiltonian for $3/2^+$ three-neutron states. The dots are the points of the rotated discretized continuum, while the circle is a three-neutron resonance. $E_r = \mathrm{Re}(E)$, $\Gamma = -2\,\mathrm{Im}(E)$.

by the usual R-matrix method that works at real energies. Then, an S matrix can be constructed from this R matrix and can be analytically continued to complex energies, and its poles can be localized [9]. We call this procedure the extended R-matrix method. So far we have implemented this method for two-cluster dynamics.

3. Application to A = 3, 4, 5, 6, and 12 nucleon systems

We briefly summarize the physics motivation and the main results of Refs. [2–6], where the methods, discussed above, were applied to selected resonances of various light nuclei. Further details can be found in the original papers. In all of our RGM calculations we used the Minnesota effective NN interaction [10] which provides a reasonably good overall description of the low-energy $N+N$ scattering phase shifts [4] and the bulk properties of the ^{3}H, ^{3}He, and ^{4}He clusters.

A=3: We searched for three-neutron resonances [4], which had been predicted from pion double charge exchange experiments on ^{3}He. All partial waves up to $J = 5/2$ were nonresonant except the $J^\pi = 3/2^+$ one, where we found a state at $E = 14$ MeV energy with 13 MeV width. The parameters of the mirror state in the three-proton system are $E = 15$ MeV and $\Gamma = 14$ MeV. Fig. 1 shows the energy eigenvalues of the complex scaled Hamiltonian for $J^\pi = 3/2^+$ three-neutron states. The three-body continuum is rotated by 2θ and is discretized because of our finite variational basis. The position of the three-neutron resonance, isolated from the continuum line, is approximately independent of θ. We have also begun a charge-independent R-matrix analysis of $N+d$ data at energies below the breakup threshold [11]. Preliminary indications from the analysis are that a number of S-matrix poles exist in the S-wave and P-wave states, some of which are (subthreshold) virtual resonances.

A=4: We studied the 0^+_2 state of ^{4}He [2], which state is notoriously difficult to reproduce in shell-models. In a $\{^3\mathrm{H}+p, {}^3\mathrm{He}+n\}$ model, which reproduced well the relevant $^1S_0$ ^{3}H$+p$ phase shift, an S-matrix pole was found corresponding to $E_r = 93$ keV and $\Gamma = 390$ keV resonance parameters. This state was also localized with $E_r = 114$ keV and $\Gamma = 392$ keV parameters in an S matrix that was constructed from a comprehensive R-matrix fit

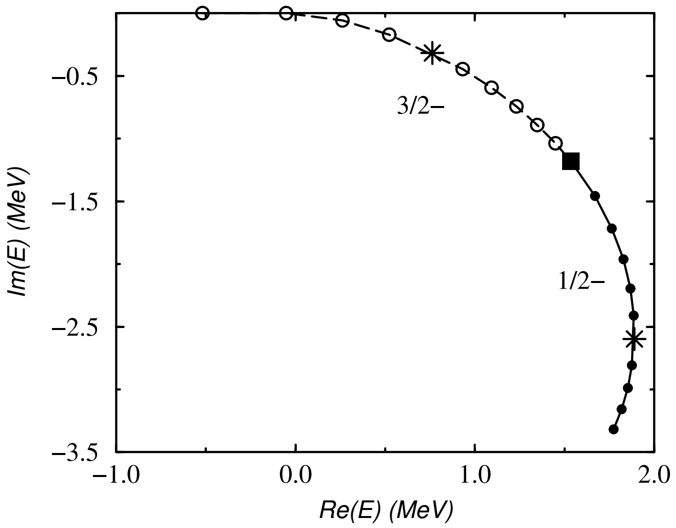

Figure 2. Trajectories of the $3/2^-$ and $1/2^-$ poles of ^{5}He as a function of the spin-orbit strength. See the text for the explanation of the symbols.

to the data. Both calculations agree that this is the first excited state of ^{4}He, and is a conventional resonance between the $^3\mathrm{H}+p$ and $^3\mathrm{He}+n$ thresholds.

A=5: We studied the low-energy ^{5}He and ^{5}Li resonances [3] with the aim to resolve the apparent contradiction between the experimentally recommended resonance parameters and the ones that appear, e.g., in the neutron halo studies of ^{6}He. Our complex S-matrix resonance parameters coming from an $\alpha+N$ RGM model and from an extended R-matrix model, respectively, were found to be in good agreement with each other. However, they differed from the conventional parameters, extracted from reaction cross sections at real energies. The most striking disagreement with the currently accepted resonance parameters [12] is in the spin-orbit splitting of the $3/2^-$ and $1/2^-$ states. Our calculations predict a much smaller splitting than given in [12]. In order to demonstrate the dependence of the $3/2^-$ and $1/2^-$ S-matrix pole positions on the spin-orbit strength ($V_{\rm SO}$) we show the ^{5}He pole trajectories in Fig. 2. The solid square shows the result for $V_{\rm SO}=0$, while the stars correspond to the physical value of the strength. By further increasing $V_{\rm SO}$, the $3/2^-$ state becomes bound. One can see that the behavior of the pole trajectory rules out the possibility that the spin-orbit splitting is larger than 2 MeV.

A=6: The low-lying three-body resonances of ^{6}He, ^{6}Li, and ^{6}Be were studied in an $\alpha+N+N$ RGM model [5]. Our motivation was to confirm or refute the existence of a new type of collective excitations, the soft dipole resonance, in the neutron halo nucleus ^{6}He. This soft dipole resonance was supposed to arise from the oscillation of the halo neutrons against the ^{4}He core. As the coupling between the core and the halo is weak, this would result in a low-energy state compared to the usual giant dipole resonances. We found all experimentally known three-body resonances of the $A=6$ nuclei, but no indication for the existence of a 1^- state in ^{6}He. Thus, using one of the most comprehensive models of that nucleus, we showed that the soft dipole resonance does not exist in ^{6}He. This result has been confirmed by other calculations in different models [13] and by a recent experiment [14].

A=12: We studied the low-lying natural-parity three-alpha resonances in ^{12}C in order to shed light on the nature of the 0_2^+ state [6]. This state, which plays an important role in stellar nucleosynthesis, was believed to be a three-alpha resonance. However, there was no unambiguous proof to support this claim. We localized all known low-energy natural-parity states of ^{12}C by using NN forces which correctly reproduced the ^{8}Be ground state resonance. We unambiguously showed for the first time in a microscopic model, that the 0_2^+ state is really a three-alpha resonance.

4. Conclusion

In summary, our study of resonances in light nuclei by means of the exploration of the analytic structure of the S matrix at complex energies has proven to be very fruitful. We observe a good general agreement between the results of our RGM and extended R-matrix models. In contrast, resonance prescriptions formulated at real energies, can lead to quite different results from one another, especially for broad states. Thus, we recommend using the complex S-pole prescription to specify resonance parameters in all cases.

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