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At the edge of nuclear stability: nonlinear quantum amplifiers

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Abstract. We show that nuclei lying at the edge of stability can behave as nonlinear quantum amplifiers. A tiny change in the nucleon-nucleon interaction can trigger a much bigger change in the binding energy of these systems, relative to the few-cluster breakup threshold.

1. Introduction

Recently, the structure and reactions of nuclei far from stability have received a lot of attention. Many of these systems possess interesting properties, such as, e.g., halos, skins, etc. Most of these unusual features are direct consequences of the very small binding energies of these nuclei, relative to the breakup into 2-3 clusters. We would like to show a hitherto unnoticed interesting property of these systems: at the edge of stability they may behave as nonlinear quantum amplifiers.

Lying very close to the breakup threshold, the constituent clusters of a nucleus far from stability are barely held together by the residual forces between them. As the energy moves closer to the breakup threshold, the nucleus becomes larger. The rate at which the nucleus moves toward breakup, as a function of its size, is determined by the rate at which the (attractive) residual interaction between the clusters vanishes, as a function of their distance. One may imagine situations (not necessarily in nuclear physics) where the residual interaction drops to zero roughly as fast as the few-cluster binding energy, while the size is increased. If this happens, then a tiny change in the basic interaction can cause only a similarly small response in the binding energy. If, however, the residual interaction between the clusters goes to zero much more slowly than the binding energy does, then tiny changes in

the basic interactions can get substantially amplified in the binding energy [1]. We demonstrate this effect through the example of the 0_2^+ state of ^{12}C.

2. The 0_2^+ state of ^{12}C as a quantum amplifier

The 0_2^+ state of ^{12}C is famous for its role played in stellar nucleosynthesis. Lying just 380 keV above the 3α threshold, this resonance is responsible for the synthesis of virtually all the carbon in the Universe through a two-step capture of three alpha particles, the so-called triple-alpha process [2]. Once ^{12}C nuclei are produced, some of them are burned further in the ^{12}C$(\alpha, \gamma)^{16}$O reaction to form ^{16}O.

We would like to see how sensitive the energy of the 0_2^+ state is to the fine details of the N-N interaction. For this purpose, we use a 12-body, 3α-cluster model of ^{12}C. The wave function of our model for ^{12}C looks like

$$\Psi^{^{12}\text{C}} = \sum_{l_1,l_2} \mathcal{A}\left\{\Phi^\alpha\Phi^\alpha\Phi^\alpha\chi^{\alpha(\alpha\alpha)}_{[l_1l_2]L}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)\right\}. \tag{1}$$

Here $\mathcal{A}$ is the intercluster antisymmetrizer, the Φ^α cluster internal states are translationally invariant $0s$ harmonic-oscillator shell-model states, the $\boldsymbol{\rho}$ vectors are the intercluster Jacobi coordinates, l_1 and l_2 are the angular momenta of the two relative motions, L is the total orbital angular momentum, and $[\ldots]$ denotes angular momentum coupling. Such a model was shown to give a good overall description of the low-lying ^{12}C states [3]. Here we follow the same method as in Ref. [3] to precisely determine the resonance energy of the 0_2^+ state.

In order to see the dependence of the results on the chosen effective N-N interaction, we performed the calculations using four different forces. The Minnesota (MN), Volkov 1 and 2 (V1, V2), and modified Hasegawa-Nagata (MHN) forces achieve similar quality in describing light nuclear systems, including ^{12}C [3]. We slightly adjusted a parameter (the exchange mixture) of each forces in order to get the 0_2^+ resonance energy right at the experimental value. The results coming from these forces are our baseline predictions. Then, we multiplied the strengths of each forces by a factor p, which varied from $p = 0.996$ to 1.004, and calculated the resonance energies again. This way we can monitor the response of the 0_2^+ state to slight perturbations in the N-N strengths.

The results for the resonance energies are shown in Table 1. As one can see, very small changes in the N-N interaction can cause almost two orders of magnitude bigger changes in the resonance energy. We believe that this is the consequence of the nonlinear amplification phenomenon discussed above. Table 1 shows that the different N-N interactions give rather different results for the dependence of the resonance energy ε on p. Simultaneous studies of the ^{8}Be ground state and the 0_2^+ state of ^{12}C indicate that the $\alpha - \alpha$ residual interaction is somewhat too strong in the case of the MHN interaction, while it is somewhat too weak in the case of the MN, V1, and V2 forces (increasingly weaker from MN to V2). Thus, we expect that the true behavior of ε with respect to small perturbations in the N-N force (as shown in Table 1) is somewhere between the MHN and MN results.

Table 1. The energy ε (in keV) of the 0_2^+ resonance, relative to the 3α threshold, as a function of the strength factor p of the various N-N interactions

	N-N interaction			
p	MHN	MN	V1	V2
1.004	235.6	273.3	294.4	306.0
1.002	308.1	327.5	337.5	343.7
1.001	344.4	353.7	358.7	361.7
1.000	379.6	379.6	379.6	379.6
0.999	414.3	405.2	400.3	397.2
0.998	448.8	430.5	420.8	414.6
0.996	517.0	481.4	460.7	450.0

3. Conclusions

The strong sensitivity of the 0_2^+ resonance energy to small changes in the N-N interaction has a spectacular consequence on the stellar production of carbon and oxygen. Since both steps of the triple-alpha reaction are governed by narrow resonances, the triple-alpha rate can be given [2], in a very good approximation, as

$$r_{3\alpha} = 3^{\frac{3}{2}} N_\alpha^3 \left(\frac{2\pi\hbar^2}{M_\alpha k_B T} \right)^3 \frac{\omega\gamma}{\hbar} \exp\left(-\frac{\varepsilon}{k_B T} \right), \qquad (2)$$

where M_α and N_α is the mass and the number density of the α particle, respectively, T is the temperature of the stellar plasma, ε is the resonance energy of the 0_2^+ state, relative to the 3α threshold, and $\omega\gamma$ is the resonance strength. Since $r_{3\alpha}$ depends exponentially on ε, even small variations in ε can cause large changes in $r_{3\alpha}$. We calculated the $r_{3\alpha}$ rates for the resonance energies shown in Table 1, and used these rates in a contemporary stellar model code [4] to see how much the tiny changes in the N-N force can influence the synthesis of carbon and oxygen [5]. We performed calculations for low-mass, medium-mass, and massive stars. The resulting carbon and oxygen abundances, relative to the standard abundances, are shown in Fig. 1 as a function of the change in the strength of the MHN and MN forces. One can easily understand the qualitative behavior of the results by noticing that a stronger force leads to a smaller ε, thus to a larger $r_{3\alpha}$, which results in a more effective triple-alpha process, relative to the $^{12}C(\alpha, \gamma)^{16}O$ burning. A similar but reversed reasoning holds for a weaker force.

The really spectacular feature of Fig. 1 is that the tiny changes in the N-N force can cause enormous changes in the carbon and oxygen abundances. At the very root of this behavior lies the nonlinear amplification phenomenon discussed above. We can say that a 0.5% change in the N-N force would lead to a situation where there is virtually no carbon or oxygen present, which would make carbon-based life impossible. As the strength of the N-N force is connected, through the pion mass, to the quark masses and ultimately to the vacuum expectation value of the Higgs
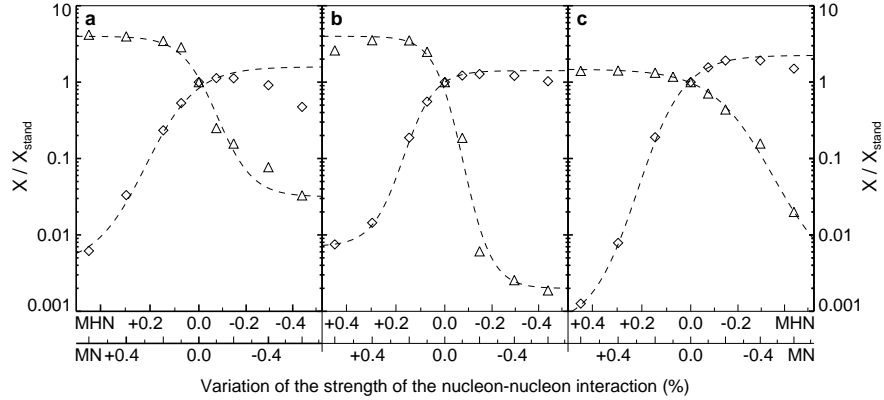
Fig. 1. The change of the carbon ($\triangle$) and oxygen ($\diamond$) mass abundances (X) through variations of the strength of the strong interaction. They are shown in panels a, b, and c for stars with masses of 20, 5, and $1.3M_\odot$, respectively, in units of the standard values X_{stand}. The variations of the strength of the strong interaction are given for the two effective N-N forces MHN and MN. The dashed curves are drawn to guide the eye.

field, our results can in principle be used to give constraints on some fundamental parameters of the Standard Model [6].

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