At the edge of nuclear stability: nonlinear quantum amplifiers

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We show that nuclear states lying at the edge of stability may show enormously enhanced response to small perturbations. For example, a 0.1% change in the strength of the strong nucleon-nucleon interaction can cause almost a hundred times bigger change in the resonance energy of the 0_2^+ state of ^{12}C.

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I. INTRODUCTION

Recently, the birth of radioactive nuclear beams has made it possible to perform extensive studies of the structure and reactions of nuclei lying at the edge of stability. It was found that these nuclei posses some really unusual properties, such as, for example, halo structure [1], which were largely unknown in stable systems. Here we would like to show that some nuclear systems close to the point of stability may exhibit a hitherto unrecognized interesting phenomenon, which we call nonlinear quantum amplifying. As the system moves toward the edge of stability, its response to small perturbations can become hugely amplified. This effect may find interesting applications in halo nuclei or in astrophysical processes.

We consider nuclei which have strong 2-body or 3-body clustering nature. This means that their wave functions contain 2- or 3-cluster structures with large weight, therefore the most important degrees of freedom are the relative motions between the clusters. For instance, the low-lying states of ^{7}Li and ^{12}C are known to have strong ^{4}He + ^{3}H and ^{4}He + ^{4}He + ^{4}He cluster structures, respectively. In addition, we assume that these systems are below or slightly above the threshold of the lowest possible breakup channel. If the energy of such a state, relative to the 2-body or 3-body breakup threshold, is close to zero then one may call it a weakly coupled few-body system. It is either a bound state or a long-lived resonance. In any case, the system is barely held together by the residual interactions acting between the clusters.

It is interesting to see how the residual interaction between the clusters behaves as the system goes from a weakly-bound state through the breakup point to a resonance state. While this happens, the size of the system increases. If the interaction between the clusters is very short-ranged, then one can imagine that the residual interaction drops down to zero as fast as the binding energy itself. Then nothing special happens. However, one can imagine situations where the residual force only slowly decreases but does not vanish, while the binding energy becomes zero. If this happens, then the binding energy becomes very sensitive to small perturbations, for example, in the nucleon-nucleon (N-N) force.

In order to see if such situations are really realized, we study three nuclear systems: the deuteron, the ground state of ^{7}Li, and the 0_2^+ state of ^{12}C.

II. RESULTS

We first choose the simplest possible example, the deuteron. The bound-state problem of the deuteron is considered in the case of a modern realistic interaction, the Reid93 force of the Nijmegen group [2]. The Schrödinger equation is solved by using the method discussed in Ref. [3]. In order to see how the response of the deuteron to small perturbations in the force would change if its binding energy were less than 2.22 MeV, we create several artificial deuterons by changing the N-N force. We multiply the strengths of each component of the N-N force (central, spin-orbit, and tensor) by $p(<1)$, and solve the Schrödinger equation for each p value. The binding energy as a function of the radius of the deuteron is shown by the dashed line in Fig. 1. The last deuteron which is generated has a 16.48 fm radius and 0.0201 MeV binding energy. In this case, all strengths in the Reid93 force are multiplied by $p = 0.82$. For a weaker interaction, the deuteron becomes unbound.

For each artificial deuteron, we calculate its response to small perturbations in the N-N force. Namely, we calculate the difference between the energies of the deuteron corresponding to a p value, and another one which comes from a 0.1% stronger N-N force. We note, that this response is closely related to the effective residual force between the clusters (between the proton and the neutron in the case of the deuteron). This response is shown by the dotted curve in Fig. 1. One can observe that the dotted curve goes down somewhat more slowly than the dashed curve, as the radius increases. As the R/B ratio shows (solid line), close to the point of instability the response can be rather large, compared to the binding energy itself. The energy of a deuteron which is bound by only 20 keV would change by 9% in response to a 0.1% change in the strength of the N-N force.

We performed several test calculations using the deuteron model. In heavier systems, discussed below, the use of realistic interactions is not feasible in our model, and effective forces have to be considered. We checked that such forces produce virtually the same result as shown in Fig. 1. First we tested the Eikemeier-Hackenbroich (EH) interaction [4], which gives a rather good overall description of the $N+N$ scattering states and of the deuteron properties, but has a Gaussian asymptotic behavior, instead of a Yukawa tail. Our second effective interaction is even much simpler. The Minnesota (MN) force [5] is designed to give the correct physical deuteron energy in a $^3S_1$ model, without tensor coupling. Yet, even this extremely simple interaction gives practically the same result as the EH or Reid93 forces. We note that in the case of the EH interaction, the deuteron becomes unbound at roughly the same $p \approx 0.82$ strength value as found in the case of the Reid93 force, while the MN interaction needs a stronger reduction, $p \approx 0.72$. The fact that we get practically the same results for rather different forces means, that the effect we see using effective forces is not an artifact related to some shortcomings of those interactions. In the following discussions we will use the MN force.

In heavier systems we cannot use the procedure of Ref. [3] to solve the Schrödinger equation. Instead, we use a variational expansion of the relative-motion wave functions in terms of Gaussian basis functions. We use this technique both for bound states and narrow (low-energy) resonances. For the unbound states this procedure is an approximation, which, however, should work rather well for narrow states. In the case of ^{7}Li, we checked that this is really the case. In the case of the deuteron, the variational procedure gives the same result as the method of Ref. [3]. Of course in the case of the very extended artificial deuteron with small binding energy, our basis have to go out to very large radii.

The second nucleus we study is ^{7}Li at its ground state, which is known to have a strong two-body, ^{4}He + ^{3}H, clustering nature. Therefore, the most important degrees of freedom is the relative motion between ^{4}He and ^{3}H. In accordance with this fact, we describe ^{7}Li by using a microscopic cluster model, similar to, e.g., Ref. [6]. The wave function of ^{7}Li looks like

$$\Psi^J = \mathcal{A}\Big\{[\Phi^\alpha\Phi^t]_S\chi_L^{\alpha t}(\boldsymbol{\rho})\Big\}, \qquad (1)$$

where $\mathcal{A}$ is the intercluster antisymmetrizer, $S=1/2$ is the intrinsic spin, $L=1$ is the relative orbital angular momentum, $J=3/2$ is the total angular momentum, the Φ^α and Φ^t cluster internal states ($\alpha = {}^4$He and $t = {}^3$H) are simple harmonic-oscillator shell-model functions, while χ is the wave function of the relative motion.

The results of our calculations for ^{7}Li are shown in Fig. 2. One can observe a markedly different behavior of the binding energy as a function of the radius than it was in the case of the deuteron. As the ^{4}He + ^{3}H two-body system is charged, it breaks up at a small radius, and the ^{7}Li bound state quickly becomes a resonance. We note in passing, that we always modify only the strong forces, while leaving the Coulomb force intact. This way we can test the sensitivity of the system at the edge of stability to small perturbations in the strong coupling constant.

One can see in Fig. 2 that while the dashed curve drops to zero very rapidly, the dotted curve of the response goes down rather slowly. As a consequence, in the vicinity of the breakup point the response to small perturbations in the N-N force gets hugely amplified.

We checked how well the pseudo-bound-state approximation used by us works for the narrow ^{7}Li resonances shown in Fig. 2. For this purpose, we localized these states as poles of the scattering matrices, using the method discussed in Ref. [7]. As expected, the results coming from the bound-state approximation are all close to those coming from the correct resonance description.

In both cases studied so far, the real physical system (shown by the black dots on the figures) was relatively far from the point where the large amplification phenomenon can appear. Our last example shows a case where the real nuclear state lies close to the strong amplification region.

The 0_2^+ state of ^{12}C is situated only 380 keV above the 3α-threshold, which makes this level one of the most important nuclear states in astrophysics. Almost all carbon in the Universe is synthesized through this state in red giant stars [8]. We use a three-cluster description of ^{12}C, similar to that in Ref. [9]. The three-alpha wave function is given as

$$\Psi^J = \mathcal{A}\Big\{\Phi^\alpha\Phi^\alpha\Phi^\alpha\chi_{[l_1l_2]L}^{\alpha(\alpha\alpha)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2)\Big\}, \qquad (2)$$

where $l_1 = l_2 = L = J = 0$, and we concentrate on the second 0^+ state.

We performed the same kind of calculations for this state as for the deuteron and ^{7}Li. The results are shown in Fig. 3. One can see that the general behavior of the binding/resonance energy and the response is qualitatively the same as in the case of ^{7}Li. However, this time the real physical position of the state is really close to the interesting region. We find that a 0.1% change in the strength of the strong N-N force leads to a roughly 7% change in the resonance energy of the physical 0_2^+ state of ^{12}C in our model. This sensitivity to the N-N force has a spectacular consequence in astrophysical carbon synthesis. Careful studies of all the details of the process show that a mere 0.5% change in the strength of the N-N force would lead to a Universe where there is virtually no carbon or oxygen present [10]. This makes carbon production one of the most fine-tuned processes in astrophysics, leading to interesting consequences for the possible values of some fundamental parameters of the Standard Model [11].

III. CONCLUSIONS

We have shown that certain nuclear states lying at the edge of stability may behave as nonlinear amplifiers: a tiny change in the N-N interaction can get enormously amplified in the binding energy or resonance energy. This behavior is mainly caused by the fact that the residual interaction between the 2-3 clusters of the nucleus goes down to zero more mildly than the binding energy itself.

We can envision several possible applications of the nonlinear amplification process discussed here. One application was exemplified through the 0_2^+ state of ^{12}C. Because of the amplification phenomenon, some astrophysical processes can be very strongly fine-tuned. Halo nuclei are also natural candidates to show this phenomenon, as the halo effect itself is strongly connected to the disappearing binding energy.

In summary, we have presented an interesting property of some nuclear systems lying at the edge of stability. Whether one can find any useful applications of this feature, beyond the one example of the 0_2^+ state of ^{12}C, remains to be seen.

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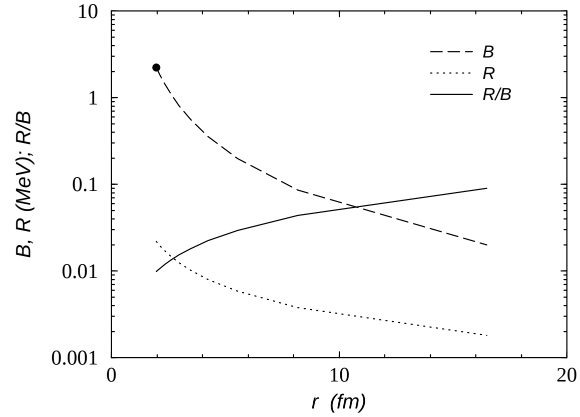

FIG. 1. The energy ($B = |E|$, where E is the binding energy or resonance energy, relative to the breakup threshold), the response ($R = |E_p - E_{p\times 1.001}|$, where E_p and $E_{p\times 1.001}$ are the binding energies or resonance energies corresponding to a given N-N force and another one which is stronger by 0.1%, respectively), and the R/B ratio calculated for several artificial deuterons, as functions of the radius of the deuteron. The N-N interaction is chosen to be the Reid93 force [2] in each case, with the strengths multiplied by a number p (see the text). The black dot shows the real physical deuteron, given by our model.

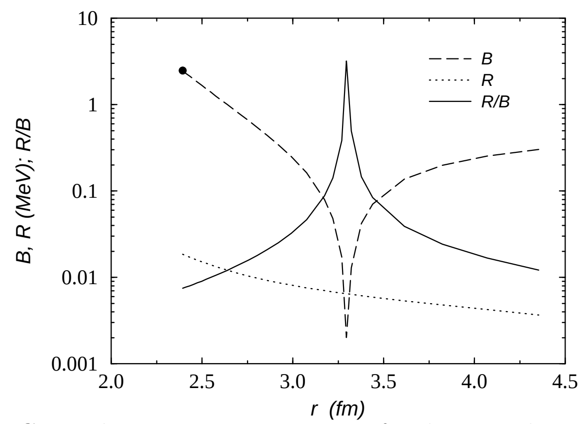

FIG. 2. The same as Fig. 1, except for the ground state of ^{7}Li. The interaction is chosen to be the MN force [5].

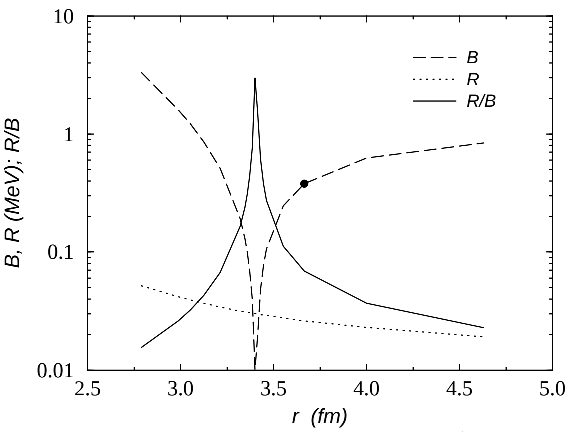

FIG. 3. The same as Fig. 2, except for the 0_2^+ state of ^{12}C. Note that the vertical scale is different from those in Figs. 1 and 2.